OPTICAL TABLES AND DATA

FOR THE USE OF OPTICIANS

SILVANUS P. THOMPSON

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BY

SILVANUS P. THOMPSON, D.Sc. F.R.S.

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OPTICAL TABLES.

1 Squares, Cubes, Roots, Reciprocals, Inverse Squares, and Logarithms of Numbers 1 to 200.

Number.	Square.	Cube.	Square Root.	Cube Root.	Reciprocal.	Inverse Square.	Logarithm.
n		n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\frac{1}{n^2}$	$\log n$
0	0	0	0·0000	0·0000	∞	∞	− ∞
1	1	1	1·0000	1·0000	1·00000	1·00000	0·0000
2	4	8	1·4142	1·2599	0·50000	0·25000	0·3010
3	9	27	1·7321	1·4422	0·33333	·11111	0·4771
4	16	64	2·0000	1·5874	0·25000	·06250	0·6021
5	25	125	2·2361	1·7100	0·20000	·04000	0·6990
6	36	216	2·4495	1·8171	0·16667	·02778	0·7782
7	49	343	2·6458	1·9129	0·14286	·02048	0·8451
8	64	512	2·8284	2·0000	0·12500	·01562	0·9031
9	81	729	3·0000	2·0801	0·11111	·01234	0·9542
10	100	1000	3·1623	2·1544	0·10000	·01000	1·0000
11	121	1331	3·3166	2·2240	0·09091	0·0082645	1·0414
12	144	1728	3·4641	2·2894	0·08333	69444	1·0792
13	169	2197	3·6056	2·3513	0·07692	59172	1·1139
14	196	2744	3·7417	2·4101	0·07143	51020	1·1461
15	225	3375	3·8730	2·4662	0·06667	44444	1·1761
16	256	4096	4·0000	2·5198	0·06250	39062	1·2041
17	289	4913	4·1231	2·5713	0·05882	34602	1·2304
18	324	5832	4·2426	2·6207	0·05556	30864	1·2553
19	361	6859	4·3589	2·6684	0·05263	27701	1·2788
20	400	8000	4·4721	2·7144	0·05000	25000	1·3010
21	441	9261	4·5826	2·7589	0·04762	0·0022675	1·3222
22	484	10648	4·6904	2·8020	0·04545	20661	1·3424
23	529	12167	4·7958	2·8439	0·04348	18903	1·3617
24	576	13824	4·8990	2·8845	0·04167	17361	1·3802
25	625	15625	5·0000	2·9240	0·04000	16000	1·3979
26	676	17576	5·0990	2·9625	0·03846	14791	1·4150
27	729	19683	5·1962	3·0000	0·03704	13717	1·4314
28	784	21952	5·2915	3·0366	0·03571	12755	1·4472
29	841	24389	5·3852	3·0723	0·03448	11891	1·4624
30	900	27000	5·4772	3·1072	0·03333	11111	1·4771

Squares, Cubes, Roots, Reciprocals, Inverse Squares, and Logarithms of Numbers 1 to 200.

Number.	Square.	Cube.	Square Root.	Cube Root.	Reciprocal.	Inverse Square.	Logarithm.
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\frac{1}{n^2}$	$\log n$
31	961	29791	5·5678	3·1414	0·03226	0·00010406	1·4914
32	1024	32768	5·6569	3·1748	0·03125	0·00097656	1·5051
33	1089	35937	5·7446	3·2075	0·03030	91827	1·5185
34	1156	39304	5·8310	3·2396	0·02941	86505	1·5315
35	1225	42875	5·9161	3·2711	0·02857	81633	1·5441
36	1296	46656	6·0000	3·3019	0·02778	77160	1·5563
37	1369	50653	6·0828	3·3322	0·02703	73046	1·5682
38	1444	54872	6·1644	3·3620	0·02632	69252	1·5798
39	1521	59319	6·2450	3·3912	0·02564	65746	1·5911
40	1600	64000	6·3246	3·4200	0·02500	62500	1·6021
41	1681	68921	6·4031	3·4482	0·02439	0·00059488	1·6128
42	1764	74088	6·4807	3·4760	0·02381	56689	1·6332
43	1849	79507	6·5574	3·5034	0·02326	54083	1·6335
44	1936	85184	6·6332	3·5303	0·02273	51653	1·6435
45	2025	91125	6·7082	3·5569	0·02222	49383	1·6532
46	2116	97336	6·7823	3·5830	0·02174	47259	1·6628
47	2209	103823	6·8557	3·6088	0·02128	45269	1·6721
48	2304	110592	6·9282	3·6342	0·02083	43403	1·6812
49	2401	117649	7·0000	3·6593	0·02041	41649	1·6902
50	2500	125000	7·0711	3·6840	0·02000	40000	1·6990
51	2601	132651	7·1414	3·7084	0·01961	0·00038447	1·7076
52	2704	140608	7·2111	3·7325	0·01923	36982	1·7160
53	2809	148877	7·2801	3·7563	0·01887	35510	1·7243
54	2916	157464	7·3485	3·7798	0·01852	34293	1·7324
55	3025	166375	7·4162	3·8030	0·01818	33058	1·7404
56	3136	175616	7·4833	3·8259	0·01786	31888	1·7482
57	3249	185193	7·5498	3·8485	0·01754	30779	1·7559
58	3364	195112	7·6158	3·8709	0·01724	29726	1·7634
59	3481	205379	7·6811	3·8930	0·01695	28727	1·7709
60	3600	216000	7·7460	3·9149	0·01667	27778	1·7782
61	3721	226981	7·8102	3·9365	0·01639	0·00026874	1·7853
62	3844	238328	7·8740	3·9579	0·01613	26014	1·7924
63	3969	250047	7·9373	3·9791	0·01587	25195	1·7993
64	4096	262144	8·0000	4·0000	0·01563	24414	1·8062
65	4225	274625	8·0623	4·0207	0·01538	23669	1·8129
66	4356	287496	8·1240	4·0412	0·01515	22957	1·8195
67	4489	300763	8·1854	4·0615	0·01493	22277	1·8261
68	4624	314432	8·2462	4·0817	0·01471	21626	1·8325
69	4761	328509	8·3066	4·1016	0·01449	21004	1·8388
70	4900	343000	8·3666	4·1213	0·01429	20408	1·8451

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\frac{1}{n^2}$	$\log n$
71	5041	357911	8·4261	4·1408	0·01408	0·00019837	1·8513
72	5184	373248	8·4853	4·1602	0·01389	19290	1·8573
73	5329	389017	8·5440	4·1793	0·01370	18765	1·8633
74	5476	405224	8·6033	4·1983	0·01351	18261	1·8692
75	5625	421875	8·6603	4·2172	0·01333	17778	1·8751
76	5776	438976	8·7178	4·2358	0·01316	17313	1·8808
77	5929	456533	8·7750	4·2543	0·01299	16866	1·8865
78	6084	474552	8·8318	4·2727	0·01282	16436	1·8921
79	6241	493039	8·8882	4·2908	0·01266	16023	1·8976
80	6400	512000	8·9443	4·3089	0·01250	15625	1·9031
81	6561	531441	9·0000	4·3267	0·01235	0·00015242	1·9085
82	6724	551368	9·0554	4·3445	0·01220	14872	1·9138
83	6889	571787	9·1104	4·3621	0·01205	14512	1·9191
84	7056	592704	9·1652	4·3795	0·01190	14172	1·9243
85	7225	614125	9·2195	4·3968	0·01176	13841	1·9294
86	7396	636056	9·2736	4·4140	0·01163	13521	1·9345
87	7569	658503	9·3274	4·4310	0·01149	13212	1·9395
88	7744	681472	9·3808	4·4480	0·01136	12913	1·9445
89	7921	704969	9·4340	4·4647	0·01124	12625	1·9494
90	8100	729000	9·4868	4·4814	0·01111	12346	1·9542
91	8281	753571	9·5394	4·4979	0·01099	0·00012076	1·9590
92	8464	778688	9·5917	4·5144	0·01087	11815	1·9638
93	8649	804357	9·6437	4·5307	0·01075	11562	1·9685
94	8836	830584	9·6954	4·5468	0·01064	11317	1·9771
95	9025	857375	9·7468	4·5629	0·01053	11080	1·9777
96	9216	884736	9·7980	4·5789	0·01042	10851	1·9823
97	9409	912673	9·8489	4·5947	0·01031	10628	1·9868
98	9604	941192	9·8995	4·6104	0·01020	10412	1·9912
99	9801	970299	9·9499	4·6261	0·01010	10203	1·9956
100	10000	1000000	10·0000	4·6416	0·01000	10000	2·0000
101	10201	1030301	10·0499	4·6570	0·00990	0·00009803	2·0043
102	10404	1061208	10·0995	4·6723	0·00980	9612	2·0086
103	10609	1092727	10·1489	4·6875	0·00971	9426	2·0128
104	10816	1124864	10·1980	4·7027	0·00962	9245	2·0170
105	11025	1157625	10·2470	4·7177	0·00952	9070	2·0212
106	11236	1191016	10·2956	4·7326	0·00943	8900	2·0253
107	11449	1225043	10·3441	4·7475	0·00935	8734	2·0294
108	11664	1259712	10·3923	4·7622	0·00926	8573	2·0334
109	11881	1295029	10·4403	4·7769	0·00917	8417	2·0374
110	12100	1331000	10·4881	4·7914	0·00909	8333	2·0414
111	12321	1367631	10·5357	4·8059	0·00901	0·00008116	2·0453
112	12544	1404928	10·5830	4·8203	0·00893	7972	2·0492
113	12769	1442897	10·6301	4·8346	0·00885	7831	2·0531
114	12996	1481544	10·6771	4·8488	0·00877	7696	2·0569
115	13225	1520875	10·7238	4·8629	0·00870	7560	2·0607
116	13456	1560896	10·7703	4·8770	0·00862	7432	2·0645
117	13689	1601613	10·8167	4·8910	0·00855	7305	2·0682
118	13924	1643032	10·8628	4·9049	0·00847	7182	2·0719
119	14161	1685159	10·9087	4·9187	0·00840	7062	2·0755
120	14400	1728000	10·9545	4·9324	0·00833	6944	2·0792

Squares, Cubes, Roots, Reciprocals, Inverse Squares, and Logarithms of Numbers 1 to 200.

Number.	Square.	Cube.	Square Root.	Cube Root.	Reciprocal.	Inverse Square.	Logarithm.
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\frac{1}{n^2}$	$\log n$
121	14641	1771561	11·0000	4·9461	0·00826	0·00006830	2·0828
122	14884	1815848	11·0454	4·9597	0·00820	6719	2·0864
123	15129	1860867	11·0905	4·9732	0·00813	6610	2·0899
124	15376	1906624	11·1355	4·9866	0·00806	6561	2·0934
125	15625	1953125	11·1803	5·0000	0·00800	6404	2·0969
126	15876	2000376	11·2250	5·0133	0·00794	6299	2·1004
127	16129	2048383	11·2694	5·0265	0·00787	6200	2·1038
128	16384	2097152	11·3137	5·0397	0·00781	6104	2·1072
129	16641	2146689	11·3578	5·0528	0·00775	6009	2·1106
130	16900	2197000	11·4018	5·0658	0·00769	5917	2·1139
131	17161	2248091	11·4455	5·0788	0·00763	0·00005827	2·1173
132	17424	2299968	11·4891	5·0916	0·00758	5739	2·1206
133	17689	2352637	11·5326	5·1045	0·00752	5653	2·1239
134	17956	2406104	11·5758	5·1172	0·00746	5568	2·1271
135	18225	2460375	11·6190	5·1299	0·00741	5487	2·1303
136	18496	2515456	11·6619	5·1426	0·00735	5407	2·1335
137	18769	2571353	11·7047	5·1551	0·00730	5328	2·1367
138	19044	2628072	11·7473	5·1676	0·00725	5251	2·1399
139	19321	2685619	11·7898	5·1801	0·00719	5175	2·1430
140	19600	2744000	11·8322	5·1925	0·00714	5102	2·1461
141	19881	2803221	11·8743	5·2048	0·00709	0·00005030	2·1492
142	20164	2863288	11·9164	5·2171	0·00704	4959	2·1523
143	20449	2924207	11·9583	5·2293	0·00699	4890	2·1553
144	20736	2985984	12·0000	5·2415	0·00694	4822	2·1584
145	21025	3048625	12·0416	5·2536	0·00690	4756	2·1614
146	21316	3112136	12·0830	5·2656	0·00685	4691	2·1644
147	21609	3176523	12·1244	5·2776	0·00680	4628	2·1673
148	21904	3241792	12·1655	5·2896	0·00676	4565	2·1703
149	22201	3307949	12·2066	5·3015	0·00671	4504	2·1732
150	22500	3375000	12·2474	5·3133	0·00667	4444	2·1761
151	22801	3442951	12·2882	5·3251	0·00662	0·00004386	2·1790
152	23104	3511808	12·3288	5·3368	0·00658	4328	2·1818
153	23409	3581577	12·3693	5·3485	0·00654	4272	2·1847
154	23716	3652264	12·4097	5·3601	0·00649	4217	2·1875
155	24025	3723875	12·4499	5·3717	0·00645	4162	2·1903
156	24336	3796416	12·4900	5·3832	0·00641	4109	2·1931
157	24649	3869893	12·5300	5·3947	0·00637	4057	2·1959
158	24964	3944312	12·5698	5·4061	0·00633	4006	2·1987
159	25281	4019679	12·6095	5·4175	0·00629	3956	2·2014
160	25600	4096000	12·6491	5·4288	0·00625	3906	2·2041

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	$\frac{1}{n^2}$	$\log n$
161	25921	4173281	12·6886	5·4401	0·00621	0·00003858	2·2068
162	26244	4251528	12·7279	5·4514	0·00617	3810	2·2095
163	26569	4330747	12·7671	5·4626	0·00613	3764	2·2122
164	26896	4410944	12·8062	5·4737	0·00610	3718	2·2148
165	27225	4492125	12·8452	5·4848	0·00606	3673	2·2175
166	27556	4574296	12·8841	5·4959	0·00602	3629	2·2201
167	27889	4657463	12·9228	5·5069	0·00599	3586	2·2227
168	28224	4741632	12·9615	5·5178	0·00595	3543	2·2253
169	28561	4826809	13·0000	5·5288	0·00592	3501	2·2279
170	28900	4913000	13·0384	5·5397	0·00588	3460	2·2304
171	29241	5000211	13·0767	5·5505	0·00585	0·00003420	2·2330
172	29584	5088448	13·1149	5·5613	0·00581	3380	2·2355
173	29929	5177717	13·1529	5·5721	0·00578	3341	2·2380
174	30276	5268024	13·1909	5·5828	0·00575	3303	2·2405
175	30625	5359375	13·2288	5·5934	0·00571	3266	2·2430
176	30976	5451776	13·2665	5·6041	0·00568	3228	2·2455
177	31329	5545233	13·3041	5·6147	0·00565	3192	2·2480
178	31684	5639752	13·3417	5·6252	0·00562	3156	2·2504
179	32041	5735339	13·3791	5·6357	0·00559	3121	2·2529
180	32400	5832000	13·4164	5·6462	0·00556	3086	2·2553
181	32761	5929741	13·4536	5·6567	0·00552	0·00003052	2·2577
182	33124	6028568	13·4907	5·6671	0·00549	3019	2·2601
183	33489	6128487	13·5277	5·6774	0·00546	2986	2·2625
184	33856	6229504	13·5647	5·6877	0·00543	2954	2·2648
185	34225	6331625	13·6015	5·6980	0·00541	2922	2·2672
186	34596	6434856	13·6382	5·7083	0·00538	2891	2·2695
187	34969	6539203	13·6748	5·7185	0·00535	2860	2·2718
188	35344	6644672	13·7113	5·7287	0·00532	2829	2·2742
189	35721	6751269	13·7477	5·7388	0·00529	2799	2·2765
190	36100	6859000	13·7840	5·7489	0·00526	2770	2·2788
191	36481	6967871	13·8203	5·7590	0·00524	0·00002741	2·2810
192	36864	7077888	13·8564	5·7690	0·00521	2713	2·2833
193	37249	7189057	13·8924	5·7790	0·00518	2685	2·2856
194	37636	7301384	13·9284	5·7890	0·00515	2657	2·2878
195	38025	7414875	13·9642	5·7989	0·00513	2630	2·2900
196	38416	7529536	14·0000	5·8088	0·00510	2603	2·2923
197	38809	7645373	14·0357	5·8186	0·00508	2577	2·2945
198	39204	7762392	14·0712	5·8285	0·00505	2551	2·2967
199	39601	7880599	14·1067	5·8383	0·00503	2525	2·2989
200	40000	8000000	14·1421	5·8480	0·00500	2500	2·3010

Logarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	8	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
100	0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	0	1	1	2	2	3	3	3	4

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′	
0°	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15	89
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15	88
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15	87
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15	86
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15	85
5	0871	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14	84
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14	83
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14	82
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14	81
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14	80
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14	79
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14	78
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14	77
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14	76
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14	75
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14	74
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14	73
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14	72
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14	71
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14	70
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14	69
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14	68
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14	67
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14	66
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13	65
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13	64
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13	63
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13	62
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13	61
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13	60
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13	59
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12	58
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12	57
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12	56
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12	55
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12	54
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12	53
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12	52
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11	51
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11	50
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11	49
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11	48
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11	47
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11	46
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10	45
45°	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10	44
	60′	54′	48′	42′	36′	30′	24′	18′	12′	6′	1′	2′	3′	4′	5′	

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1′	2′	3′	4′	5′	
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10	43
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10	42
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10	41
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9	40
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9	39
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9	38
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9	37
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9	36
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8	35
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8	34
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8	33
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8	32
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8	31
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7	30
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7	29
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7	28
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7	27
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6	26
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6	25
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6	24
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6	23
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6	22
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5	21
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5	20
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5	19
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5	18
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	4	4	17
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4	16
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4	15
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4	14
76	9703	9707	9711	9716	9720	9724	9728	9732	9736	9740	1	1	2	3	3	13
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3	12
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3	11
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3	10
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2	9
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2	8
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2	7
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2	6
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2	5
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1	4
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1	3
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1	2
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0	1°
89	9998	9999	9999	9999	9999	9999	9999	9999	9999	9999	0	0	0	0	0	0
90	1·000	..	..	..	..	..	..	..	..	..			..			
	60′	54′	48′	42′	36′	30′	24′	18′	12′	6′	1′	2′	3′	4′	5′	

Natural Tangents.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	14
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	6	10	13	17
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	10	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	10	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29
45°	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30

Natural Tangents.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	26	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	23	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	$\bar{0}057$	$\bar{0}145$	$\bar{0}233$	$\bar{0}323$	$\bar{0}413$	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	74	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	118
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	130
71	2·9042	9208	9375	9544	9714	9887	0061	0237	0415	0595	29	58	87	115	144
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	162	203
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972					
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4·7046	7453	7867	8288	8716	9152	9594	$\bar{0}045$	$\bar{0}504$	$\bar{0}970$					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	0405	$\bar{1}066$	$\bar{1}742$	$\bar{2}432$					
81	6·3138	3859	4596	5350	6122	6912	7920	8548	9395	0264					
82	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	$\bar{0}285$					
83	8·1443	2636	3863	5126	6427	7769	9152	0579	$\bar{2}052$	$\bar{3}572$					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
90	∞	..	..	..	..	..	..	..	..	..					

Versed Sines.

Angle $\theta°$.	Versin θ.	Angle $\theta°$.	Versin θ.	Angle $\theta°$.	Versin θ.	Angle $\theta°$.	Versin θ.	Angle $\theta°$.	Versin θ.
·5°	0·0000	18·5	0·0517	36·5	0·1961	54·5	0·4193	72·5	0·6993
1°	0·0002	19	0·0545	37	0·2014	55	0·4264	73	0·7076
1·5°	0·0003	19·5	0·0574	37·5	0·2066	55·5	0·4336	73·5	0·7160
2°	0·0006	20	0·0603	38	0·2120	56	0·4408	74	0·7244
2·5	0·0010	20·5	0·0633	38·5	0·2174	56·5	0·4481	74·5	0·7328
3	0·0014	21	0·0664	39	0·2229	57	0·4554	75	0·7412
3·5	0·0019	21·5	0·0696	39·5	0·2284	57·5	0·4627	75·5	0·7496
4	0·0024	22	0·0728	40	0·2340	58	0·4701	76	0·7581
4·5	0·0031	22·5	0·0761	40·5	0·2396	58·5	0·4775	76·5	0·7666
5	0·0038	23	0·0795	41	0·2453	59	0·4850	77	0·7750
5·5	0·0046	23·5	0·0829	41·5	0·2510	59·5	0·4925	77·5	0·7836
6	0·0055	24	0·0865	42	0·2569	60	0·5000	78	0·7921
6·5	0·0064	24·5	0·0900	42·5	0·2627	60·5	0·5076	78·5	0·8006
7	0·0075	25	0·0937	43	0·2686	61	0·5152	79	0·8092
7·5	0·0086	25·5	0·0974	43·5	0·2746	61·5	0·5228	79·5	0·8178
8	0·0097	26	0·1012	44	0·2807	62	0·5305	80	0·8264
8·5	0·0110	26·5	0·1051	44·5	0·2867	62·5	0·5383	80·5	0·8350
9	0·0123	27	0·1090	45	0·2929	63	0·5460	81	0·8436
9·5	0·0137	27·5	0·1130	45·5	0·2991	63·5	0·5538	81·5	0·8522
10	0·0152	28	0·1171	46	0·3053	64	0·5616	82	0·8608
10·5	0·0167	28·5	0·1212	46·5	0·3116	64·5	0·5695	82·5	0·8695
11	0·0184	29	0·1254	47	0·3180	65	0·5774	83	0·8781
11·5	0·0201	29·5	0·1296	47·5	0·3244	65·5	0·5853	83·5	0·8868
12	0·0219	30	0·1340	48	0·3309	66	0·5933	84	0·8955
12·5	0·0237	30·5	0·1384	48·5	0·3374	66·5	0·6013	84·5	0·9042
13	0·0256	31	0·1428	49	0·3439	67	0·6093	85	0·9128
13·5	0·0276	31·5	0·1474	49·5	0·3506	67·5	0·6173	85·5	0·9215
14	0·0297	32	0·1520	50	0·3572	68	0·6254	86	0·9302
14·5	0·0319	32·5	0·1566	50·5	0·3639	68·5	0·6335	86·5	0·9390
15	0·0341	33	0·1613	51	0·3707	69	0·6416	87	0·9477
15·5	0·0364	33·5	0·1661	51·5	0·3775	69·5	0·6498	87·5	0·9564
16	0·0387	34	0·1710	52	0·3843	70	0·6580	88	0·9651
16·5	0·0412	34·5	0·1759	52·5	0·3912	70·5	0·6662	88·5	0·9738
17	0·0437	35	0·1808	53	0·3982	71	0·6744	89	0·9825
17·5	0·0463	35·5	0·1859	53·5	0·4052	71·5	0·6827	89·5	0·9913
18	0·0489	36	0·1910	54	0·4122	72	0·6910	90	1·0000

6 Vulgar Fractions Converted to Decimals, with their Square Roots and Cube Roots.

Fraction.	Decimal.	Square Root.	Cube Root.	Fraction.	Decimal.	Square Root.	Cube Root.
$\frac{1}{2}$	0·5000	0·707	0·794	$\frac{3}{8}$	0·3750	0·612	0·721
$\frac{1}{3}$	0·3333	0·577	0·693	$\frac{5}{8}$	0·6250	0·791	0·855
$\frac{2}{3}$	0·6667	0·816	0·874	$\frac{7}{8}$	0·8750	0·935	0·956
$\frac{1}{4}$	0·2500	0·500	0·630	$\frac{1}{9}$	0·1111	0·333	0·481
$\frac{3}{4}$	0·7500	0·866	0·909	$\frac{2}{9}$	0·2222	0·471	0·606
$\frac{1}{6}$	0·1667	0·408	0·550	$\frac{4}{9}$	0·4444	0·667	0·763
$\frac{5}{6}$	0·8333	0·913	0·941	$\frac{5}{9}$	0·5556	0·745	0·822
$\frac{1}{7}$	0·1428	0·378	0·523	$\frac{7}{9}$	0·7778	0·882	0·920
$\frac{2}{7}$	0·2857	0·535	0·659	$\frac{8}{9}$	0·8889	0·943	0·961
$\frac{3}{7}$	0·4286	0·655	0·754	$\frac{1}{12}$	0·0833	0·289	0·437
$\frac{4}{7}$	0·5714	0·756	0·830	$\frac{5}{12}$	0·4167	0·645	0·747
$\frac{5}{7}$	0·7143	0·845	0·894	$\frac{7}{12}$	0·5833	0·764	0·836
$\frac{6}{7}$	0·8571	0·926	0·950	$\frac{11}{12}$	0·9166	0·957	0·971
$\frac{1}{8}$	0·1250	0·354	0·500	$\frac{1}{15}$	0·0667	0·258	0·405

7 Inch-Fractions Converted into Decimals.

$\frac{1}{64}$	0·015625	$\frac{17}{64}$	0·265625	$\frac{33}{64}$	0·515625	$\frac{49}{64}$	0·765625
$\frac{1}{32}$	0·03125	$\frac{9}{32}$	0·28125	$\frac{17}{32}$	0·53125	$\frac{25}{32}$	0·78125
$\frac{3}{64}$	0·046875	$\frac{19}{64}$	0·296875	$\frac{35}{64}$	0·546875	$\frac{51}{64}$	0·796875
$\frac{1}{16}$	0·0625	$\frac{5}{16}$	0·3125	$\frac{9}{16}$	0·5625	$\frac{13}{16}$	0·8125
$\frac{5}{64}$	0·078125	$\frac{21}{64}$	0·328125	$\frac{37}{64}$	0·578125	$\frac{53}{64}$	0·828125
$\frac{3}{32}$	0·09375	$\frac{11}{32}$	0·34375	$\frac{19}{32}$	0·59375	$\frac{27}{32}$	0·84375
$\frac{7}{64}$	0·109375	$\frac{23}{64}$	0·359375	$\frac{39}{64}$	0·609375	$\frac{55}{64}$	0·859375
$\frac{1}{8}$	0·125	$\frac{3}{8}$	0·375	$\frac{5}{8}$	0·625	$\frac{7}{8}$	0·875
$\frac{9}{64}$	0·140625	$\frac{25}{64}$	0·390625	$\frac{41}{64}$	0·640625	$\frac{57}{64}$	0·890625
$\frac{5}{32}$	0·15625	$\frac{13}{32}$	0·40625	$\frac{21}{32}$	0·65625	$\frac{29}{32}$	0·90625
$\frac{11}{64}$	0·171875	$\frac{27}{64}$	0·421875	$\frac{43}{64}$	0·671875	$\frac{59}{64}$	0·921875
$\frac{3}{16}$	0·1875	$\frac{7}{16}$	0·4375	$\frac{11}{16}$	0·6875	$\frac{15}{16}$	0·9375
$\frac{13}{64}$	0·203125	$\frac{29}{64}$	0·453125	$\frac{45}{64}$	0·703125	$\frac{61}{64}$	0·953125
$\frac{7}{32}$	0·21875	$\frac{15}{32}$	0·46875	$\frac{23}{32}$	0·71875	$\frac{31}{32}$	0·96875
$\frac{15}{64}$	0·234375	$\frac{31}{64}$	0·484375	$\frac{47}{64}$	0·734375	$\frac{63}{64}$	0·984375
$\frac{1}{4}$	0·25	$\frac{1}{2}$	0·5	$\frac{3}{4}$	0·75		

8 Inches and Decimals of 1 Inch to Millimetres.

Ins.	0	·1	·2	·3	·4	·5	·6	·7	·8	·9
0	0·00	2·54	5·08	7·62	10·16	12·70	15·24	17·78	20·32	22·86
1	25·40	27·94	30·48	33·02	35·56	38·10	40·64	43·18	45·72	48·26
2	50·80	53·34	55·88	58·42	60·96	63·50	66·04	68·58	71·12	73·66
3	76·20	78·74	81·28	83·82	86·36	88·90	91·44	93·98	96·52	99·06
4	101·6	104·14	106·68	109·22	111·76	114·30	116·84	119·38	121·92	124·46
5	127·0	129·54	132·08	134·62	137·16	139·70	142·24	144·78	147·32	149·86
6	152·4	154·94	157·48	160·02	162·56	165·10	167·64	170·18	172·72	175·26
7	177·8	180·34	182·88	185·42	187·96	190·50	193·04	195·58	198·12	200·66
8	203·2	205·74	208·28	210·82	213·36	215·90	218·44	220·98	223·52	226·06
9	228·6	231·14	233·68	236·22	238·76	241·30	243·84	246·38	248·92	251·46
10	254·0	256·54	259·08	261·62	264·16	266·70	269·24	271·78	274·32	276·86
11	279·4	281·94	284·48	287·02	289·56	292·10	294·64	297·18	299·72	302·26

Inches and Sixteenths to Millimetres.

Ins.	0	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$
0	0·0	1·59	3·18	4·76	6·35	7·94	9·53	11·11	12·70	14·29	15·88	17·46	19·05	20·64	22·23	23·81
1	25·4	26·99	28·57	30·16	31·75	33·34	34·92	36·51	38·10	39·69	41·27	42·86	44·45	46·04	47·62	49·21
2	50·8	52·39	53·97	55·56	57·15	58·74	60·32	60·91	63·50	65·09	66·67	68·26	69·85	71·44	73·02	74·61
3	76·2	77·79	79·37	80·96	82·55	84·14	85·72	87·31	88·90	90·49	92·07	93·66	95·25	96·84	98·42	100·0
4	101·6	103·1	104·8	106·4	108·0	109·5	111·1	112·7	114·3	115·9	117·5	119·1	120·7	122·2	123·8	125·4
5	127·0	128·6	130·2	131·8	133·4	134·9	136·5	138·1	139·7	141·3	142·9	144·5	146·1	147·6	149·2	150·8
6	152·4	154·0	155·6	157·2	158·8	160·5	161·9	163·5	165·1	166·7	168·3	169·9	171·5	173·0	174·6	176·2
7	177·8	179·4	181·0	182·6	184·2	185·7	187·3	188·9	190·5	192·1	193·7	195·3	196·9	198·0	200·1	201·6
8	203·2	204·8	206·4	208·0	209·6	211·1	212·7	214·3	215·9	217·5	219·1	220·7	222·3	223·8	225·4	227·0
9	228·6	230·2	231·8	233·4	235·0	236·5	238·1	239·7	241·3	242·9	244·5	246·1	247·7	249·2	250·8	252·4
10	254·0	255·6	257·2	258·8	260·4	261·9	263·5	265·1	266·7	268·3	269·9	271·5	273·1	274·6	276·2	277·8
11	279·4	281·0	282·6	284·2	285·7	287·3	288·9	290·5	292·1	293·7	295·3	296·9	298·6	300·0	301·6	303·1

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Feet and Inches to Millimetres.

Feet.	Inches.											
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
0	0·00	25·4	50·8	76·2	101·6	127·0	152·4	177·8	203·2	228·6	254·0	279·4
1	304·8	330·2	355·6	381·0	406·4	431·8	457·2	482·6	508·0	533·4	558·8	584·2
2	609·6	635·0	660·4	685·8	711·2	736·6	762·0	787·4	812·8	838·2	863·6	889·0
3	914·4	939·8	965·2	990·6	1016·0	1041·4	1066·8	1092·2	1117·6	1143·0	1168·4	1193·8
4	1219·2	1244·6	1270·0	1295·4	1320·8	1346·2	1371·6	1397·0	1422·4	1447·8	1473·2	1498·6
5	1524·0	1549·4	1574·8	1600·2	1625·6	1651·0	1676·4	1701·8	1727·2	1752·6	1778·0	1803·4
6	1828·8	1854·2	1879·6	1905·0	1930·4	1955·8	1981·2	2006·6	2032·0	2057·4	2082·8	2108·2
7	2133·6	2159·0	2184·4	2209·8	2235·2	2260·6	2286·0	2311·4	2236·8	2362·2	2387·6	2413·0
8	2438·4	2463·8	2489·2	2514·6	2540·0	2565·4	2590·8	2616·2	2641·6	2667·0	2692·4	2717·8
9	2743·2	2768·6	2794·0	2819·4	2844·8	2870·2	2895·6	2921·0	2946·4	2971·8	2997·2	3022·6

Millimetres to Inches.

Milli-metres.	0	1	2	3	4	5	6	7	8	9
0	0·0000	0·03937	0·07874	0·11811	0·15748	0·19685	0·23622	0·27560	0·31497	0·35434
10	·3937	0·43307	0·47244	0·51181	0·55118	0·59055	0·62992	0·66930	0·70867	0·74804
20	·7874	0·82677	0·86614	0·90551	0·94488	0·98425	1·02362	1·06300	1·10237	1·13174
30	1·1811	1·22047	1·25984	1·29921	1·33858	1·37795	1·41732	1·45670	1·49607	1·53544
40	1·5748	1·61417	1·65354	1·69291	1·73228	1·77165	1·81102	1·85040	1·88977	1·92914
50	1·9685	2·00787	2·04724	2·08661	2·12598	2·16535	2·20472	2·24410	2·28347	2·32284
60	2·3622	2·40157	2·44094	2·48031	2·51968	2·55905	2·59842	2·63780	2·67717	2·71654
70	2·7560	2·79537	2·83474	2·87411	2·91348	2·95285	2·99222	3·03160	3·07097	3·11034
80	3·1497	3·18907	3·22844	3·26781	3·30718	3·34655	3·38592	3·42530	3·46467	3·50404
90	3·5434	3·58277	3·62214	3·66151	3·70088	3·74025	3·77962	3·81900	3·85837	3·89774

The term *micron*, and symbol μ, are used by microscopists to mean $\frac{1}{1000}$ of a millimetre or 10^{-6} metre. Similarly, the symbol $\mu\mu$ is used to mean $\frac{1}{1000}$ of a micron or $\frac{1}{1000000}$ of a millimetre or 10^{-9} metre.

Millimetres to Inches.

Milli-metres.	0	10	20	30	40	50	60	70	80	90
0	0·0000	0·3937	0·7874	1·1811	1·5748	1·9685	2·3622	2·7560	3·1497	3·5434
100	3·937	4·331	4·724	5·118	5·512	5·906	6·299	6·693	7·087	7·480
200	7·874	8·268	8·661	9·055	9·449	9·843	10·236	10·630	11·024	11·317
300	11·811	12·205	12·598	12·992	13·386	13·780	14·173	14·567	14·961	15·354
400	15·748	16·142	16·535	16·929	17·323	17·717	18·110	18·504	18·898	19·291
500	19·685	20·079	20·472	20·866	21·260	21·654	22·047	22·441	22·835	23·228
600	23·622	24·016	24·409	24·803	25·197	25·591	25·984	26·378	26·772	27·165
700	27·560	27·954	28·347	28·741	29·135	29·529	29·922	30·316	30·710	31·103
800	31·497	31·891	32·284	32·678	33·072	33·466	33·859	34·253	34·647	35·040
900	35·434	35·828	36·221	36·615	37·009	37·403	37·796	38·190	38·584	38·977

13 Comparison of Metric and British Units.

LENGTH.

1 Metre = 1000 Millimetres = 39·37043 Inches.
1 Millimetre = 0·001 Metre = 0·03937 Inches.
1 Inch = 0·0254 Metre = 25·4 Millimetres.
1 Foot = 0·3048 Metre = 304·8 Millimetres.

AREA.

1 Square Millimetre = 0·00155 Square Inch.
1 Square Inch = 645·1 Square Millimetres.

MASS.

1 Kilogramme = 1000 Grammes = 2·2046 Pounds = 15432·2 Grains.
1 Gramme = 0·001 Kilogramme = 15·432 Grains.
1 Pound = 7000 Grains = 16 Ounces = 453·6 Grammes.
1 Ounce = 437·5 Grains = 28·35 Grammes.
1 Grain = 0·0648 Gramme.
N.B.—The legal Grain in Great Britain is $\frac{1}{7000}$ of 1 Pound, and is the same Grain in Troy weight as in Avoirdupois weight.

JEWELLERS' WEIGHTS.

1 Ounce (Troy) = 480 Grains = 31·1035 Grammes.
" " = 151·5 Diamond Carats = 606 Diamond Grains.
1 Diamond Carat = 4 Diamond Grains = 3·1683 Grains = 0·2053 Gramme.
1 Diamond Grain = 0·792 Grains = 0·051325 Gramme.
1 Gramme = 19·483 Diamond Grains = 4·871 Diamond Carats.

14 Velocity of Light.

The velocity of Light *in vacuo* appears to be almost exactly 300,000 kilometres per second, or 186,400 miles per second. In air it is about 80 kilometres or 50 miles per second slower. In water, glass and other dense media the velocity is less, and differs for different colours, the velocity being less for violet and blue light than it is for red and orange light. The following are velocities for yellow light in several media :—

Velocity in	Kilometres per Second.	Miles per Second.	Velocity relatively to that in Air.	Slowness relatively to that in Air = μ.
Air	300000	186400	1	1
Water ..	225000	139800	0·75	1·3337
Crown glass	196080	121830	0·6536	1·53
Flint glass	184050	114350	0·6135	1·63

N.B.—The slowness of light in any medium, relatively to that in air, is called "the refractive index" of that medium. It is the reciprocal of the velocity.

WAVE-LENGTHS AND FREQUENCIES.

	Region of Spectrum.	Name of Line.	Element.	Wave-length. micro-millimetres.	Wave-length. millionths of inch.	Frequency (billions per second).
Invisible.	Infra-red part of Spectrum, or Heat-waves.	Rubens' & Nichols' longest waves		24000	944	12·5($\times 10^{12}$)
		Langley's longest waves		15000	592	20
		Paschen's longest waves		9450	370	31·7
		ψ_2	..	2700	106·24	111
		ψ_1	..			
		Φ_2	..	1240	48·73	242
		Φ_1	..	1200	47·25	250
		Y	..	899·04	35·36	333·7
			..	898·65	35·35	334·0
		X_4	..	880·61	34·64	340·8
		X_3	..	866·14	34·1	346·2
		X_2	..	854·18	33·63	351·3
		X_1	..	849·7	33·44	353·3
		Z	..	822·64	32·34	364·5
VISIBLE SPECTRUM.	Red . .	A	O	759·4	29·28	395·2
		B	O	686·74	27·03	436·5
	Orange .	— C	H	656·30	25·83	457·2
		D_1	Na	589·61	23·21	508·8
	Yellow .	D_2	Na	589·02	23·18	509·1
		D_3	He	587·60	23·13	510·5
	Green .	E_1	Fe	527·05	20·78	569·2
			Ca	527·04	20·78	569·2
		E_2	Fe	526·97	20·74	569·3
	Peacock .	b_1	Mg	518·38	20·40	578·9
		b_2	Mg	517·29	20·36	580·0
		b_3	Fe	516·92	20·351	580·4
			Fe	516·91	20·350	580·4
		b_4	Fe	516·77	20·306	580·5
			Mg	516·75	20·305	580·5
	Blue . .	— F	H	486·15	19·14	617·1
		G	Fe	430·81	16·96	696·3
			Ca	430·79	16·95	696·4
		h	H	410·18	16·17	731·3
	Violet .	H	Ca	396·86	15·63	756·0
		K	Ca	393·38	15·48	762·7
Invisible (Photographic).	Ultra-violet part of Spectrum, or Actinic Waves. — Produces Photographic effects and excites both Fluorescence and Phosphorescence.	L	Fe	382·06	15·04	785·1
		M	Fe	372·78	14·676	804·6
			Fe	372·71	14·673	804·9
		N	Fe	358·13	14·09	837·7
		O	Fe	344·11	13·55	871·8
		P	Fe	336·13	13·23	892·6
		Q	Fe	328·69	12·94	912·6
		R	Ca	318·14	12·52	942·9
			Ca	317·94	12·51	943·5
		r	Fe	314·46	12·38	954·1
		S_1	Fe	310·08	12·207	967·4
			Fe	310·04	12·206	967·6
		S_2	Fe	310·01	12·205	967·7
		ε	Fe	304·77	11·99	984·5
		T	Fe	302·12	11·894	993·0
			Fe	302·07	11·892	993·3
		t	Fe	299·45	11·79	1002·0
		U	Fe	294·80	11·60	1017·6
		Miller's limit, photographic		202	7·95	1485·1
		Stokes' limit, fluorescent		185	7·28	1621·6
		Schumann's highest frequency		100	3·93	3000

16 JENA GLASS. Selected Sorts arranged in order of refractivity for equal mean dispersion.

Factory Numbers.	Description.	Density.	Refractive Index for D.	Partial Dispersion.			Medium Dispersion, C to F.	$\nu = \frac{\mu - 1}{\Delta \mu}$
				A^1 to D.	D to F.	F to G^1.		
O 225	Light phosphate crown	2·58	1·5159	0·00485	0·00515	0·00407	0·00737	70·0
S 40	Medium phosphate crown	3·07	1·5590	0·00546	0·00587	0·00466	0·00835	66·9
S 30	Dense barium phosphate crown	3·35	1·5760	0·00570	0·00622	0·00500	0·00884	65·2
O 802	Boro-silicate crown (very light)	2·38	1·4967	0·00504	0·00534	0·00423	0·00765	64·9
O 144	" "	2·47	1·5100	0·00519	0·00559	0·00446	0·00797	64·0
O 599	" "	2·48	1·5069	0·00529	0·00569	0·00457	0·00813	62·3
O 57	Light silicate crown	2·46	1·5086	0·00530	0·00578	0·00464	0·00823	61·8
O 40	Silicate crown	2·49	1·5166	0·00545	0·00596	0·00479	0·00849	60·9
O 337	" "	2·60	1·5144	0·00547	0·00596	0·00480	0·00847	60·7
O 374	" "	2·48	1·5109	0·00547	0·00593	0·00479	0·00844	60·5
O 546	Zinc crown	2·59	1·5170	0·00555	0·00605	0·00485	0·00859	60·2
O 60	Calcium silicate crown	2·49	1·5179	0·00553	0·00605	0·00487	0·00860	60·2
O 138	Silicate crown of high refractive index .	2·53	1·5285	0·00560	0·00614	0·00494	0·00872	60·2
O 567	Silicate crown	2·51	1·5134	0·00554	0·00605	0·00488	0·00859	59·7
O 20	Silicate crown of low refractive index . . .	2·47	1·5019	0·00543	0·00592	0·00478	0·00842	59·6
O 227	Barium silicate crown	2·73	1·5399	0·00582	0·00639	0·00514	0·00909	59·4
O 203	Ordinary silicate crown	2·54	1·5175	0·00563	0·00616	0·00499	0·00877	59·0
O 610	Crown of low refractivity for yellow light . .	2·51	1·5063	0·00552	0·00602	0·00489	0·00858	59·0
O 598	Silicate crown	2·59	1·5152	0·00562	0·00619	0·00499	0·00879	58·6
O 512	" "	2·64	1·5195	0·00568	0·00625	0·00504	0·00886	58·6
O 13	Potassium silicate crown	2·53	1·5228	0·00572	0·00637	0·00515	0·00901	58
O 15	Zinc silicate crown	2·74	1·5308	0·00587	0·00644	0·00520	0·00915	58
O 211	Dense barium silicate crown	3·21	1·5726	0·00630	0·00702	0·00568	0·00995	57·5
O 709	Zinc soda crown	2·58	1·5128	0·00575	0·00630	0·00508	0·00894	57·3
O 1209	Densest baryta crown	3·55	1·6112	0·00680	0·00753	0·00610	0·01068	57·2
O 153	Silicate crown	2·53	1·5160	0·00576	0·00637	0·00516	0·00904	57·2
O 114	Soft silicate crown	2·55	1·5151	0·00577	0·00642	0·00521	0·00910	56·6
O 197	Boro-silicate glass	2·64	1·5250	0·00599	0·00654	0·00531	0·00929	56·5
O 463	Baryta light flint	3·11	1·5646	0·00648	0·00720	0·00586	0·01020	55·4
O 608	Crown of high dispersion	2·60	1·5149	0·00595	0·00666	0·00543	0·00943	54·6
O 722	Baryta light flint	3·26	1·5797	0·00681	0·00761	0·00621	0·01078	53·8
O 602	" " "	3·12	1·5676	0·00675	0·00759	0·00618	0·01072	53·0
O 846	" " "	3·01	1·5525	0·00657	0·00736	0·00602	0·01042	53·0

JENA GLASS. Selected Sorts arranged in order of refractivity for equal mean dispersion—*continued*

Factory Numbers.	Description.	Density.	Refractive Index for D.	Partial Dispersion.			Medium Dispersion, C to F.	$\nu = \frac{\mu - 1}{\Delta \mu}$
				A[1] to D.	D to F.	F to G[1].		
O 381	Crown of high dispersion	2·70	1·6262	0·00644	0·00727	0·00596	0·01026	51·3
O 152	Silicate glass	2·76	1·5368	0·00659	0·00743	0·00610	0·01049	51·2
O 683	Baryta light flint	3·16	1·5688	0·00696	0·00786	0·00644	0·01110	51·2
O 643	„ „ „	3·11	1·5637	0·00699	0·00790	0·00650	0·01115	50·6
O 527	„ „ „	3·19	1·5718	0·00706	0·00803	0·00660	0·01133	50·4
O 164	Boro-silicate flint	2·81	1·6503	0·00710	0·00786	0·00644	0·01114	49·4
O 575	Baryta light flint	3·15	1·5682	0·00718	0·00817	0·00672	0·01151	49·3
O 214	Silicate glass	2·73	1·5366	0·00690	0·00781	0·00644	0·01102	48·7
O 522	Baryta light flint	3·03	1·5554	0·00718	0·00819	0·00677	0·01153	48·2
O 726	Extra light flint	2·87	1·5398	0·00711	0·00810	0·00669	0·01142	47·3
O 161	Boro-silicate flint	2·97	1·5676	0·00762	0·00860	0·00709	0·01216	46·7
O 678	Baryta light flint	3·29	1·5825	0·00777	0·00891	0·00739	0·01255	46·4
O 378	Extra light flint	2·93	1·5473	0·00739	0·00847	0·00705	0·01193	45·9
O 154	Light silicate flint	3·16	1·5710	0·00819	0·00943	0·00791	0·01327	43·0
O 376	Ordinary light flint	3·12	1·5660	0·00814	0·00939	0·00787	0·01319	42·9
O 230	Silicate flint with relatively high refractive index	3·40	1·6014	0·00868	0·01009	0·00843	0·01415	42·5
O 340	Ordinary light flint	3·21	1·5774	0·00857	0·00994	0·00837	0·01396	41·4
O 569	„ „ „	3·22	1·5738	0·00853	0·00987	0·00831	0·01385	41·4
O 184	Light silicate flint	3·28	1·5900	0·00882	0·01022	0·00861	0·01438	41·1
O 748	Baryta flint	3·67	1·6235	0·00965	0·01142	0·00965	0·01599	39·1
O 318	Ordinary light flint	3·48	1·6031	0·00960	0·01124	0·00952	0·01576	38·3
O 118	Ordinary silicate flint	3·58	1·6129	0·01006	0·01184	0·01008	0·01660	36·9
O 167	„ „ „	3·60	1·6169	0·01026	0·01206	0·01029	0·01691	36·5
O 103	„ „ „	3·63	1·6202	0·01034	0·01220	0·01041	0·01709	36·2
O 93	„ „ „	3·68	1·6245	0·01053	0·01243	0·01063	0·01743	35·8
O 226	„ „ „	3·72	1·6287	0·01072	0·01270	0·01086	0·01776	35·4
O 335	Dense silicate flint	3·77	1·6372	0·01099	0·01308	0·01124	0·01831	34·8
O 102	„ „ „	3·87	1·6489	0·01152	0·01372	0·01180	0·01919	33·8
O 192	„ „ „	4·10	1·6734	0·01255	0·01507	0·01302	0·02104	32·0
O 41	„ „ „	4·49	1·7174	0·01439	0·01749	0·01521	0·02434	29·5
O 113	„ „ „	4·64	1·7371	0·01526	0·01870	0·01632	0·02600	28·4
O 165	„ „ „	4·78	1·7541	0·01607	0·01974	0·01730	0·02743	27·5
O 198	Very dense silicate flint	4·99	1·7782	0·01719	0·02120	0·01868	0·02941	26·5
S 67	Densest silicate flint	6·33	1·9626	0·02767	0·03547	0·03252	0·04882	19·7

Compiled from the circulars of Messrs. Schott & Co., excluding those kinds of glass which experience has shown to be liable to deteriorate in time. The line A[1] is the potassium (red) line of $\lambda = 76·77$ microcentimetres, and the line G[1] the hydrogen line of $\lambda = 43·41$, respectively near to Frauenhofer's lines A and G.

17 Composition of Some Specimens of Jena Glass
(from Pulfrich).

Light Phosphate Crown.

O 225 $\mu_D = 1{\cdot}5160$ $\nu = 70{\cdot}3$	P_2O_5	70·5
	K_2O	12
	Al_2O_3	10
	MgO, B_2O_3, As_2O_5	7·5

Phosphate Crown.

S 40 $\mu_D = 1{\cdot}5619$ $\nu = 66{\cdot}5$	P_2O_5	59·5
	BaO	28·0
	DiO	3·0
	B_2O_3, As_2O_5	9·5

Boro-silicate Crown.

O 627 $\mu_D = 1{\cdot}5128$ $\nu = 63{\cdot}7$	SiO_2	68·24
	B_2O_3	10·00
	K_2O	9·50
	Na_2O	10·00
	ZnO	2·00
	Mn_2O_3	0·07
	As_2O_5	0·20

Light Borate Crown.

S 205 $\mu_D = 1{\cdot}5075$ $\nu = 60{\cdot}6$	B_2O_3	69
	Al_2O_3	18
	Na_2O, BaO	13
	As_2O_5	traces

Heavy Barium-silicate Crown.

O 211 $\mu_D = 1{\cdot}5727$ $\nu = 58$	SiO_2	48·7
	BaO	29·0
	ZnO	10·3
	Alkalis & B_2O_3	12·0

Light Barium Flint.

O 527 $\mu_D = 1{\cdot}5718$ $\nu = 50{\cdot}6$	SiO_2	51·6
	PbO	10·0
	BaO	20·0
	ZnO	7·0
	Alkalis	11·4

Light Boro-silicate Flint.

O 658 $\mu_D = 1{\cdot}5452$ $\nu = 50{\cdot}3$	SiO_2	32·7
	B_2O_3	31·0
	PbO	25·0
	Alkalis & Al_2O_3	11·3

Light Silicate Flint.

O 154 $\mu_D = 1{\cdot}5710$ $\nu = 43{\cdot}1$	SiO_2	54·22
	B_2O_3	1·50
	PbO	33·00
	K_2O	8·00
	Na_2O	3·00
	Mn_2O_3	0·08
	As_2O_5	0·20

Heavy Silicate Flint.

O 165 $\mu_D = 1{\cdot}7545$ $\nu = 27{\cdot}6$	SiO_2	28·36
	PbO	69·0
	K_2O	2·50
	Mn_2O_3	0·04
	As_2O_5	0·10

Densest Silicate Flint.

S 57 $\mu_D = 1{\cdot}9625$ $\nu = 19{\cdot}7$	SiO_2	18
	PbO	82

CHANCE'S GLASS. Arranged in Order of Refractivity for Equal Mean Dispersion.

Factory Number.	Description of Glass.	Density.	Refractive Index for D.	Partial Dispersion.			Medium Dispersion, C to F.	$\nu = \frac{\mu_D - 1}{\Delta \mu}$
				C to D.	D to F.	F to G_1.		
B 646	Borosilicate Crown	2·45	1·5093	0·00236	0·00552	0·00449	0·00788	64·6
A 605	Hard Crown . .	2·48	1·5175	0·00252	0·00604	0·00484	0·00856	60·5
A 569	Soft „ . .	2·55	1·5152	0·00264	0·00642	0·00517	0·00906	56·9
B 565	Baryta „ . .	3·17	1·5660	0·00297	0·00709	0·00576	0·01006	56·3
B 555	Densest baryta Crown .	3·58	1·6099	0·00321	0·00779	0·00629	0·01100	55·5
B 535	Baryta light Flint .	2·94	1·5452	0·00298	0·00722	0·00582	0·01020	53·5
A 490	Extra light Flint .	2·78	1·5316	0·00313	0·00772	0·00630	0·01085	49·0
A 485	„ „ „ .	2·80	1·5333	0·00322	0·00777	0·00640	0·01099	48·5
B 468	Baryta light Flint .	3·29	1·5840	0·00362	0·00866	0·00735	0·01248	46·8
A 458	Light Flint . . .	2·93	1·5472	0·00348	0·00848	0·00707	0·01196	45·8
A 432	„ „ . . .	3·06	1·5610	0·00372	0·00927	0·00770	0·01299	43·2
A 410	„ „ . . .	3·22	1·5760	0·00402	0·01002	0·00840	0·01404	41·0
A 370	Dense Flint . .	3·57	1·6124	0·00474	0·01176	0·01030	0·01650	37·0
A 361	„ „ . . .	3·64	1·6214	0·00491	0·01231	0·01046	0·01722	36·1
A 360	„ „ . . .	3·66	1·6225	0·00493	0·01236	0·01054	0·01729	36·0
A 337	Extra dense Flint .	3·88	1·6469	0·00541	0·01376	0·01170	0·01917	33·7
A 299	Double extra dense Flint	4·40	1·7129	0·00670	0·01714	0·01661	0·02384	29·9

19 VARIOUS GLASSES. Arranged in Order of Refractivity for Equal Mean Dispersion.

Samples Examined by Baille.	Temp. °C.	Density.	Refractive Index for D line.	Partial Dispersion.			Medium Dispersion, C to F.	$\nu = \frac{\mu_D - 1}{\Delta \mu}$
				A to D.	D to F.	F to G.		
Crown Glass . .	17°·8	2·50	1·5280	..	0·0063	0·0054	0·0089	59·3
„ „ . .	23°·5	2·49	1·5160	..	0·0062	0·0056	0·0088	58·6
„ „ . .	21°·2	2·80	1·5192	..	0·0064	0·0057	0·0090	57·7
„ „ . .	18°·4	2·55	1·5265	..	0·0067	0·0060	0·0095	55·4
„ „ . .	21°·9	3·00	1·5604	..	0·0086	0·0079	0·0122	45·9
Flint Glass . . .	23°·2	2·98	1·5660	..	0·0088	0·0080	0·0124	45·6
„ „ . . .	18°·4	3·22	1·5715	..	0·0098	0·0089	0·0138	41·4
„ „ . . .	22°·0	3·24	1·5822	..	0·0102	0·0094	0·0141	41·3
„ „ . . .	19°·5	3·44	1·6027	.	0·0114	0·0105	0·0159	37·9
„ „ . . .	23°·2	3·54	1·6109	..	0·0116	0·0110	0·0163	37·5
„ „ . . .	24°·0	3·68	1·6304	..	0·0125	0·0120	0·0174	36·2
„ „ . . .	13°·7	3·63	1·6198	..	0·0123	0·0114	0·0172	36·0
„ „ . . .	12°·4	4·08	1·6858	..	0·0161	0·0152	0·0224	30·6
„ „ . . .	22°·5	5·00	1·7920	..	0·0229	0·0219	0·0318	24·5

20 The Effect of Temperature on the refractive index of glass is small, and may, as a rule, be neglected altogether. The order of magnitude of the effect is given below, from the determinations of Müller for certain specimens of crown and flint glass.

Crown Glass between Temp. $-5°$ and $+23°$ C. $\mu_D = 1{\cdot}516149 + {\cdot}00000017\ t.$

Flint Glass between „ $-3°$ and $+21°$ C. $\mu_D = 1{\cdot}579856 + {\cdot}00000323\ t.$

Pulfrich gives following as the coefficients for selected kinds of Jena Glass (see Table 16): S 40, $-0{\cdot}00000305$; O527, $+0{\cdot}0000014$; O 154, $+0{\cdot}00000261$; S 57, $+0{\cdot}00001447$. In all cases, whether the coefficient for yellow light is + or −, the *dispersion* increases with rise of temperature.

REFRACTIVE INDICES of various ISOTROPIC SOLIDS.

$\Delta\mu = \mu_F - \mu$

Substance.	Refractive Indices for the lines of the Spectrum named.								$\nu = \frac{\mu_D - 1}{\Delta\mu}$
	A	B	C	D	E	F	G	H	
Ammonium chloride	..	1·6326	1·6366	1·6422	1·6464	1·6533	1·6613	..	33·4
Amber	..	1·5418	1·5296	1·5462	1·5504	1·5543	..	..	22·1
Antimony glass	..	..	..	[2·013]	..	..	..	..	..
Arsenic tribromide	..	..	..	..	..	..	..	..	..
Arsenite	..	..	1·748	1·755	..	..	..	..	..
Barium nitrate	..	..	1·5665	1·5716	..	1·5825	..	..	35·7
Beeswax	[1·542]	..	..	..	..	..	..	..	..
Blende	..	..	[2·34165]	2·3692	..	..	..	..	..
Borax	..	..	1·5122	1·5148	..	1·5207	..	..	60·6
Boric acid	..	..	1·4624	1·4630	..	1·4702	..	..	59·4
Camphor	..	..	..	1·532	..	..	..	..	..
Caoutchouc	[1·524]	..	..	..	..	..	..	..	..
Diamond (colourless)	2·4024	2·4073	2·4100	2·4173	2·4269	2·4354	2·4514	2·4648	16·4
„ (brown)	..	[2·4606]	..	2·4699	2·4790	..	..	..	..
Ebonite	..	..	..	1·6	..	..	..	..	..
Fuchsine	1·73	1·81	1·99	..	..	..	1·31	1·54	..
Gelatine	..	..	..	1·547	..	..	..	..	..
Horn	..	..	..	1·565	..	..	..	..	..
Lead chromate	..	..	..	2·5 to 2·97	..	..	..	..	..
Mastic	[1·535]	..	..	..	..	..	..	..	..
Obsidian	..	1·4928	1·4939	1·4964	1·4994	1·5017	..	..	63·6
Pitch	[1·531]	..	..	..	..	..	..	..	..
Phosphorus	.. ..	.. ..	.. ..	2·093 } 2·144 }	2·1583 ..	.. ..	.. ..	.. ..	.. ..
Quartz (fused)	1·454	..	..	1·4585	..	1·4632	1·4669	..	67·9
Resins—									
Balsam of copaiba	..	..	..	1·549	..	..	..	..	..
Canada balsam	..	..	1·5124	1·526	..	1·5351	..	..	41·5
Copal	1·528	..	..	..	..	..	..	..	..
Mastic	1·535	..	..	..	..	..	..	..	..
Balsam of Peru	..	1·585	..	1·593	1·613	..	..	..	..
Resin (colophony)	..	..	1·545	1·548	..	..	..	..	..
Shellac	..	..	..	1·525	..	..	..	..	..
Rock salt	1·5366	1·5392	1·5405	1·5442	1·5490	1·5532	1·5613	1·5683	42·8
Selenium vitreous	2·653	2·730	2·86	2·98	..	..	..	..	..
Silver bromide	..	..	..	2·2533	..	..	..	..	..
„ chloride	..	..	..	2·061	..	..	..	..	..
„ iodide	..	..	..	2·1816	..	..	..	..	9·6
Spermaceti	[1·535]	..	..	..	..	..	..	..	..
Tallow	[1·49]	..	..	..	..	..	..	..	..

Thallium glass 2·00 on

See page at end

REFRACTIVE INDICES of Various LIQUIDS.

Name of Liquid.	Refractive Indices for the lines of the Spectrum named.										$\frac{\mu_D - 1}{\mu_F - \mu_C}$
	Temp.	A	B	C	D	E	F	G	H	$\mu_F - \mu_C$	
Acetic acid	20°	..	..	1·36985	1·37182	..	1·37648	[1·38017]	..	·00663	56·1
	20°	..	..	1·37022	1·37218	..	1·37683	[1·38057]	..	·00661	56·3
Acetone	10°	..	..	1·3626	1·3646	..	1·3694	[1·3732]	..	·0068	53·6
Alcohol (ethyllic)	0°	..	..	1·3677	1·3695	..	..	[1·3773]	..	..	..
	15°	1·3600	1·3612	1·3621	1·3638	1·3661	1·3683	[1·3720]	1·3751	·0062	58·6
Aniline	20°	..	..	1·5778	1·5863	..	1·6041	[1·6204]	..	·0048	22·1
Benzine	10°	..	..	1·4983	1·5029	..	1·5148	[1·5355]	..	·0165	30·5
	15°	1·4905	1·4937	1·4955	1·5002	1·5066	1·5124	1·5234	1·5329	·0169	29·6
Carbon bisulphide	15°	1·6114	1·6177	1·6209	1·6303	1·6434	1·6554	1·6779	1·7035	·0345	18·3
Cinnamic ether	..	1·5456	1·5507	1·5530	1·5607	1·5816	1·6038	1·6261	..	·0508	11·0
Chinolin	20°	..	..	1·6094	1·6171	..	1·6361	[1·6497]	..	·0267	23·1
Chloroform	10°	1·4438	1·4457	1·4466	1·4490	1·4526	1·4555	1·4614	1·4661	·0089	50·5
	20°	..	..	1·4437	1·4462	1·4525	..	..	..	..	..
	30°	..	..	..	1·4397	..	..	..	1·4561	..	..
Ether ethyl	15°	1·3529	1·3545	1·3554	1·3566	1·3590	1·3606	1·3646	1·3683	·0042	84·9
Glycerine	15°	1·4661	1·4677	1·4688	1·4711	1·4741	1·4766	[1·4812]	1·4853	·0078	60·3
Kreasote	..	..	1·5320	1·5335	1·5383	1·5452	1·5515	1·5639	1·5744	·0180	29·9
Metacinnamene	15°	..	..	1·592	1·597	..	1·612	[1·624]	..	·0200	29·5
Methylalcohol	15°	..	..	1·3308	1·3326	1·3	1·3362	..	1·3421	·0054	61·6
Methylene-di-iodide	15°	..	..	1·732	1·743	..	1·767	[1·794]	..	·035	21·2
Monobromnapthalin	20°	1·6405	1·6464	1·6495	1·6582	1·6705	1·6819	[1·7041]	1·7289	·0324	20·3
Naphthylphenylketone	15°	..	..	1·659	1·669	..	1·697	..	..	·038	17·6
Naphthylphenylketone-di-bromide	..	..	..	..	1·700	..	..	..	..	..	..
Phenol	..	..	..	..	1·549	..	..	..	..	..	..
Phenyl-thio-carbimide	10°	..	..	1·646	1·654	..	1·681	[1·706]	..	·035	18·7
Piperine	18°	..	..	1·665	1·681	..	1·734	[1·806]	..	·069	9·87

(26)

REFRACTIVE INDICES of Various LIQUIDS—*continued.*

Name of Liquid.	Refractive Indices for the lines of the Spectrum named.										$\frac{\mu_D - 1}{\mu_F - \mu_C}$
	Temp.	A	B 686·7	C 656·3	D 589·3	E 527	F 486·1	G 430·8	H	$\mu_F - \mu_C$	
Phosphorus in methylene-di-iodide in equal weights	18°	..	..	1·929	1·944	..	1·984	[2·021]	..	·055	17·2
Quinidine	15°	..	..	1·596	1·602	..	1·621	[1·639]	..	·025	24·1
Sulphur in methylene-di-iodide	16°	..	..	..	1·778	..	..	..	..	..	..
Toluene	20°	..	..	1·4911	1·4955	..	1·5070	[1·5170]	..	·0159	31·2
Water	16°	..	..	1·3318	1·3337	..	1·3378	1·3442	..	·0060	56·6
	18°·75	..	1·3309	1·3317	1·3336	1·3359	1·3378	1·3413	1·3442	·0061	54·7
Xylol	19°·7	1·4859	..	1·4908	1·1495	..	1·5059	[1·5153]	..	·0151	32·6
Oils (various)											
Almond oil	0°	..	..	1·4755	1·4782	..	1·4847	..	..	·0092	52·0
Aniseed oil	15°·1	..	1·5487	1·5508	1·5573	1·5659	1·5744	1·5912	1·6084	·0236	23·6
	21°·4	..	..	1·5410	1·5475	..	1·5647	..	..	·0237	23·1
Bitter almond oil	20°	..	..	1·5391	..	..	1·5623	[1·5775]	..	·0232	..
Cassia oil	10°	..	..	1·6007	1·6104	..	1·6389	..	1·7039	·0382	15·9
	15°	..	1·5659	1·5690	1·5780	1·5904	1·6029	..	..	·0339	17·0
Castor oil	..	..	..	..	1·490	..	..	..	..	..	..
Cedar-wood oil	..	..	..	..	1·510	..	..	..	..	..	..
,, ,, hardened	..	..	..	..	1·520	..	..	..	..	..	..
Cinnamon oil	23°·5	1·5967	1·6038	1·6077	1·6188	1·6348	1·6508	..	..	·0431	14·3
Clove oil	..	..	..	..	1·533	..	..	..	..	..	..
Linseed oil	..	..	..	..	1·485	..	..	..	..	..	..
Nutmeg oil	25°	1·4594	1·4655	..	..	..	..	..	..	..	..
Olive oil	0°	..	..	1·4738	1·4763	..	1·4825	..	..	·0087	54·7
Poppy oil	..	..	..	..	1·463	..	..	..	..	..	..
Rock oil (petroleum)	0°	..	..	1·4345	1·4573	..	1·4644	..	..	·0299	15·3
Turpentine	10°·6	..	1·4705	1·4715	1·4744	1·4784	1·4817	1·4882	1·4939	·0102	46·5

23 REFRACTIVE INDICES of certain CRYSTALS.

Name of Mineral.	Index of Refraction for D line.	Partial Dispersion.			Mean Dispersion, C to F.	$\frac{\mu_D - 1}{\Delta \mu}$
		A to D.	D to F.	F to G.		
Fluorspar (Mülheims)	1·43384	0·00381	0·00312	..	0·00446	97·3
„ (Gifford)	1·43385	0·00290	0·00321	0·00256	0·00454	95·5
Calcspar, ordinary ray (Mascart) . .	1·65846	0·00833	0·00947	0·00827	0·01347	48·9
„ extraordinary ray (Mascart) .	1·48654	0·00369	0·00430	0·00386	0·00610	79·8
Quartz, ordinary ray (Mascart) . .	1·54423	0·00521	0·00543	0·00463	0·00778	69·95
„ extraordinary ray (Mascart) .	1·55338	0·00526	0·00559	0·00475	0·00802	69·00
„ ordinary ray (Gifford) . .	1·54425	0·00519	0·00544	0·00427	0·00777	70·04
„ extraordinary ray (Gifford) . .	1·55337	0·00537	0·00562	0·00442	0·00804	68·82
Vitreous silica (fused quartz) (Gifford) .	1·45848	0·00459	0·00469	0·00368	0·00675	67·9

The values for quartz are those for right-handed crystals. For left-handed quartz crystals both Van der Willigen and Gifford have found the refractive indices very slightly lower. Gifford gives, for D line, ordinary index 1·5442363, extraordinary index 1·5533452.

The refractive index of fluorspar, as also those of quartz and rock-salt, decreases with an increase of temperature.

In all fluids, increase of temperature lowers the dispersion.

24 REFRACTIVE INDICES OF GASES.

Name of Gas.	Kind of Light or Line of Spectrum.	Refractive Index.
Hydrogen	D line	1·000139
Water vapour	„	1·000259
Oxygen	„	1·000271
Nitrogen	„	1·000298
Carbon monoxide	„	1·000335
Ammonia gas	„	1·000379
Marsh gas (CH_4)	„	1·000444
Carbon dioxide	„	1·000454
Mercury vapour	Red	1·000556
Olefiant gas (C_2H_4)	D line	1·000723
Chlorine	„	1·000773
Cyanogen	„	1·000822
Bromine	„	1·001132

The above results are those of Mascart, with the exception of that for Mercury vapour, which is from Le Roux. (μ for Vacuum = 1.) The most recent determinations by G. W. Walker, in *Proc. Roy. Society*, March 1903, give values mostly slightly higher. These values are for the standard pressure of 760 mm., at 0° C. At other pressures and temperatures the values of the indices for each gas vary in almost exact proportion with the density.

Refraction and Dispersion for Dry Atmospheric Air.

	Index of Refraction for D line.	Partial Dispersion.			Mean Dispersion.	$\frac{\mu_D - 1}{\Delta \mu}$
		A to D.	D to F.	F to G.		
Dry Air	1·0002922	0·0000017	0·0000021	0·0000019	0·0000029	100·7

25 Atomic Refraction.

When a substance changes its density, in consequence of change of temperature or pressure, it is found that its refractivity changes proportionally (Gladstone's law).

If ρ represent density, then

$\frac{\mu - 1}{\rho}$ = a constant, called the *specific refractivity* of the substance.

This law may be applied to calculate the refractivity of mixtures of two substances that do not chemically act on one another.

Further, the specific refractivity of a compound multiplied by its molecular equivalent is called the *molecular refractivity* of that compound; and the specific refractivity of an element multiplied by its atomic equivalent is called the *atomic refractivity* of that element. In the following Table are given the *atomic refractivities* of most of the elements on the authority of Dr. Gladstone. When compounds are formed out of the elements, the refractivity of the compound depends upon the specific refractivities of the constituent elements, and in many cases can be calculated from them. Certain substances, notably lead, thallium and phosphorus, have high atomic refractive power, and they are useful therefore as constituents of flint glass. Zinc, on the other hand, and calcium, potassium and sodium, have low atomic refractivities, and they are used as constituents in crown glasses of low refracting quality.

The difference between the atomic refractivities of an element for red and for violet light is called its *dispersion equivalent*, or its *atomic dispersivity*; the refractivity being determined for the A-line in the red, and for the H-line in the violet part of the spectrum. Gladstone has given the following values for the *dispersion equivalents* of some of the elements: Phosphorus, 3·0; Sulphur, 2·6 or 1·2; Hydrogen, 0·04; Carbon, 2·6 or 6·6; Oxygen, 0·18 or 0·10; Chlorine, 0·50; Bromine, 1·22; Iodine, 3·65; Nitrogen, 0·10.

Element.	Atomic Weight.	Specific Refractivity.	Atomic Refractivity.
Hydrogen	1·008	1·488	1·5
Lithium	7·0	0·514	3·6
Beryllium	9·0	0·733	6·6
Boron	11·0	0·436 or 0·317	4·8 or 3·5
Carbon	12·0	0·383	4·6
Nitrogen	14·03	0·343, &c.	4·8, &c.
Oxygen	16·0	0·203 or 0·169	3·25 or 2·7
Fluorine	19·0	0·031	0·6?
Argon	19·94	0·159	3·17
Sodium	23·05	0·202	4·65
Magnesium	24·3	0·287	6·9
Aluminium	27·0	0·352	9·5
Silicon	28·4	0·250 or 0·204	7·1 or 5·8
Phosphorus	31·0	0·594	18·4, &c.
Sulphur	32·0	0·422 or 0·500, &c.	13·5 or 16·0, &c.
Chlorine	35·45	0·282 or 0·302	10·0 or 10·7
Potassium	39·11	0·205	8·0
Calcium	40·0	0·252	10·1
Titanium	48·0	0·522	25·0
Vanadium	51·4	0·481	24·7?
Chromium	52·1	0·296	15·4
Manganese	55·0	0·208	11·5
Iron	56·0	0·209 or 0·355	11·7 or 19·9
Nickel	58·7	0·186	11·0
Cobalt	59·5	0·183	10·9
Copper	63·6	0·184	11·7
Zinc	65·3	0·151	9·9
Gallium	69·0	0·214	14·75
Arsenic	75·0	0·200	15·0
Selenium	79·0	0·339, &c.	26·8, &c.
Bromine	79·95	0·190 or 0·213	15·2 or 17·0
Rhubidium	85·5	0·133	11·4
Strontium	87·66	0·152	13·3
Yttrium	89·1	0·197	17·6
Zirconium	90·6	0·242	21·9
Rhodium	103·0	0·232	23·9?
Palladium	106·5	0·213	22·7
Silver	107·92	0·121	13·1
Cadmium	112·0	0·124	13·9
Indium	113·7	0·153	17·4
Tin	119·0	0·232 or 0·161	27·6 or 19·2
Antimony	120·0	0·204 or 0·200	24·5 or 24·0
Iodine	126·85	0·192 or 0·214	24·4 or 27·2
Cæsium	132·9	0·117	15·6
Barium	137·43	0·117	16·1
Lanthanum	138·2	0·143	19·8
Cerium	140·2	0·143	20·0?
Iridium	193·1	0·165	31·9?
Platinum	195·0	0·172	33·5
Gold	197·3	0·127	25·1
Mercury	200·0	0·107 or 0·099	21·5 or 19·8?
Thallium	204·0	0·106	21·6
Lead	206·95	0·129 or 0·119	26·7? or 24·5
Bismuth	208·0	0·154	32·0?
Thorium	232·6	0·123	28·7

27 THE SPHEROMETER.

Of all the instruments with which the optical constructor has to do, the SPHEROMETER is that upon which he is mainly dependent for precision. It is to him what the balance is to the chemist, and must therefore be thoroughly understood. Its object is *to measure the curvature of surfaces.* To explain curvature and its relation to focal length a word is necessary about the measurement of curvature in general. The larger the radius of a circle, the less curved will be the circle itself. A bit—say an inch in length—of a six-inch circle is less curved than a bit (equally long)—of a three-inch circle. In fact the latter, having half the radius, is exactly twice as much curved. By doubling the radius we halve the curvature. Or, generalising, the *curvature*, of a line or surface, *is inversely proportional to the radius of curvature.* In order to describe, therefore, the curvature of any line or surface, it is necessary and sufficient to state *the reciprocal of the radius* of that curved line or surface. It only remains to choose what length of radius shall be, for this purpose, taken as a standard. By international agreement the *metre* (= 1000 millimetres = 39·37 inches) has been so chosen. Hence the curve having unit curvature—one **dioptrie,** see Art. **28**, p. 36 —is a curve of one metre radius. It follows that a curvature of two dioptries will correspond to a radius of half a metre (= 500 millimetres = 19·69 inches). A curve whose radius is $\frac{1}{10}$ metre will have a curvature of ten dioptries. As the curvature of a surface is proportional to the reciprocal of its radius of curvature, the principle of the spherometer is based on the calculation of that radius from measurement made of the surface.

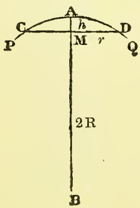

FIG. 1.

Suppose that we wished to find out the length of the radius of the arc P Q of a circle; we may proceed as follows (Fig. 1):—

Draw any chord C D. Bisect C D in M. Through M draw a perpendicular A M B, meeting the curve in A. Now we know that this line A M B passes through the centre of the circle of which P Q forms part; and that if A M B were

to meet the other side of the circle at B, A B would be a diameter.

Then

$$AM \times MB = (MD)^2$$

or

$$AM \times (AB - AM) = (MD)^2$$

If we call the radius of curvature R, then $AB = 2R$.

Let the length A M, which is called the *sagitta* of the curve, be called h, and let the length M D be called r. Substituting, we get

$$h(2R - h) = r^2,$$

whence

$$R = \frac{r^2 + h^2}{2h} \quad . \quad . \quad . \quad . \quad . \quad [1]$$

If h is *small* compared with R, this becomes

$$R = \frac{r^2}{2h} \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

whence

$$\frac{1}{R} = \frac{2h}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad [3]$$

Now imagine the arc P Q to be rotated about the axis A B as in Fig. 2. Then it will generate a spherical surface. The line C D will describe a plane, and the point D will sweep out a "small" circle, of which r is the radius. The centre of curvature O of the sphere is clearly the same as that of the arc; O A (= R) being its radius of curvature. If, then, we select two points such as C D on a spherical surface, and by some means measure their distance apart,

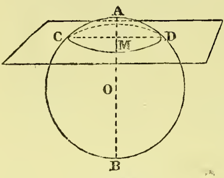

Fig 2.

and also measure the sagitta A M of the curve, we can calculate the radius of curvature of the sphere.

A spherometer is an instrument for doing this. This, in one of its simple commercial forms, is called a *lens-measure* (Figs. 8 and 9, Art. 28). The dial of the instrument is so graduated that it shall read off the curvature in *dioptries* directly.

 D

The Spherometer—*continued.*

For more accurate work a *spherometer* of greater precision, such as that depicted in Fig. 3, is needed. It has three fixed legs, so that it can stand steadily when placed on a spherical surface with these three legs alone in contact. These three feet form the corners of an equilateral triangle, or may be regarded as three equidistant points on the "small" circle C D of Fig. 2. The distance from any one of these three feet to the central leg is equal to the radius r of this circle. The fourth leg is capable of being screwed up or down by a micrometer screw, in order to measure the height h of the sagitta of the curvature of the surface. English spherometers usually have 50 threads to the inch, so that giving one turn to the central screw raises or lowers the central foot $\frac{1}{50}$ of an inch. Continental micrometers usually have a screw-thread of half a millimetre, or twenty threads to the centimetre. The head of the micrometer screw consists of a circle divided into 100 equal parts so as to enable fractions of one turn to be read off with accuracy. Sometimes the head is divided into 500 parts for greater accuracy. In the case of these spherometers with three fixed equidistant legs, the length a between any two of the legs is equal to $r\sqrt{3}$, so that the formulæ become

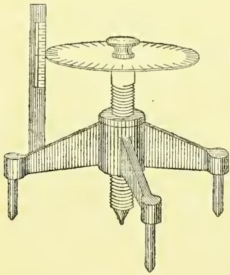

Fig. 3.

$$R = \frac{a^2 + 3h^2}{6h} \quad . \quad . \quad . \quad . \quad [1a]$$

or approximately,

$$R = \frac{a^2}{6h} \quad . \quad . \quad . \quad . \quad . \quad . \quad [2a]$$

whence

$$\frac{1}{R} = \frac{6h}{a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [3a]$$

If h, a and r are given in *millimetres*, the curvature of the surface *in dioptries* is given by the following formulæ:—

$$\frac{6000\,h}{a^2 + 3h^2}, \text{ or approximately } \frac{6000\,h}{a^2}, \text{ or } \frac{2000\,h}{r^2}.$$

Or if h, a and r are given in *inches*, the curvature *in dioptries* is—

$$\frac{236 \cdot 4\,h}{a^2 + 3\,h^2}, \text{ or approximately } \frac{236 \cdot 4\,h}{a^2}, \text{ or } \frac{78 \cdot 8\,h}{r^2}.$$

If the dioptries of curvature of the two surfaces of the lens are thus found and added algebraically together, the *power* of the lens *in dioptries* is found by multiplying by $(\mu - 1)$.

28 MODERN OPTICAL FORMULÆ, and the Principles on which they are based.

In modern optical practice, lenses are described not in terms of their *focal length*, as used to be the case, but in terms of their *power*, that is to say in terms of the convergivity or divergivity which they exercise upon the light that passes through them. Light consists of waves, which in all ordinary media, such as air or glass, always march at right angles to their own wave-fronts. If the wave-fronts are flat or "plane," then the paths of the waves are parallel. If the wave-fronts have a bulging curvature as in the light which is spreading radially from a luminous point, then the paths are divergent. If the wave-fronts have a hollow curvature, then the paths will converge to a focus. All that any lens or system of lenses can do to the waves that pass through it is to imprint upon the wave-fronts a new curvature.

A lens which is thicker in the middle than at the edges, commonly called a convex lens, when placed in the path of a parallel beam of light, causes that light to converge to a focus because it imprints a hollow curvature upon the advancing wave-fronts. This it does, because, owing to the slower speed of light when travelling through glass, that part of the wave-front which passes through the thickest part of the lens is retarded most. Convex lenses, which thus have a positive power or convergivity because they concentrate the waves at a *real* or positive focus, are accordingly specified as *positive* lenses or *plus* (+) lenses; whilst those lenses,—commonly called concave lenses, which are thinner in the middle than at the edges,—imprint a bulging curvature on the wave-front, causing the waves

to diverge as from a negative or *virtual* focus. Hence such lenses, having a negative power or convergivity are properly described as *negative* lenses, or *minus* (–) lenses.

In order to be able to prescribe lenses of the required degree of power, the system of numbering lenses according to their convergivity—i.e. their power to imprint a curvature on the wave-surface of the light—has been adopted by international agreement dating from 1879. In this international system the unit of curvature chosen is named one **Dioptrie** (sometimes spelled *Dioptre*).

We may apply this principle to illustrate some of the well-known formulæ used in lens calculations. It is well-known that the focal length f of a lens, depends partly on the refractivity of the glass of which it is made, and partly on the curvatures of its two surfaces. Suppose that both the surfaces are convex, and that one of them has radius r_1 and the other radius r_2. Then the curvature of each surface is found by taking the reciprocal of the radius. The curvature of the first surface is therefore $\frac{1}{r_1}$, and that of the second surface $\frac{1}{r_2}$. Adding these together we get for the total curvature $\frac{1}{r_1} + \frac{1}{r_2}$. This must then be multiplied by the refractivity of the glass of the kind used. Table **16**, p. 20, gives values of the *refractive index* μ of various glasses. Subtracting 1 (i.e. the refractivity of air) from the refractive index, gives us $\mu - 1$ the *refractivity* in air of glass of that kind. If we multiply the total curvature of lens by $\mu - 1$, we shall get as the product the curvature that the lens will imprint on the light that passes through it. This curvature will be the reciprocal of its focal length or $\frac{1}{f}$. Putting all these things together into one formula we have

$$\frac{1}{f} = (\mu - 1) \times \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad . \quad . \quad . \quad . \quad (1)$$

Example.—Let a convex lens have as its two radii of curvature 0·050 metre and 0·200 metre, and let it be made of glass having mean refractive index 1·52. Then $(\mu - 1)$ will be 0·52, $r_1 = 0{\cdot}050$, and $r_2 = 0{\cdot}200$. Then $\frac{1}{r_1} = 20$ dioptries, $\frac{1}{r_2} = 5$ dioptries; total curvature = 25 dioptries.

Multiplying by 0·52 we get 13 dioptries as the convergivity or power of the lens. The focal length is therefore $\frac{1}{13}$ metre (= 0·0769 metre or 3·03 inches).

N.B.—The curvature of a convex surface is reckoned *positive*, that of a hollow or concave face, *negative*. In the case of any lens that has a concave face with radius of curvature r, the curvature of that face would be reckoned as $-\frac{1}{r}$.

A further illustration is afforded by the calculation of the distances of conjugate foci. Suppose light to be proceeding from a point P on the left. Its wave-surface as it spreads will diverge, the radius of it being the distance back to the point P. Call

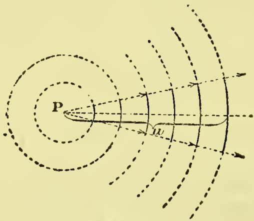

Fig. 4.

this distance u. At that distance from P the wave-surface will have a curvature $\frac{1}{u}$, which will become smaller as the distance increases. Thus if distance u reckoned back to P is 200 milli-

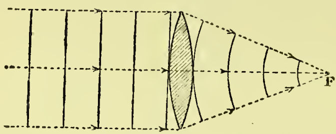

Fig. 5.

metres or 0·200 metre, the curvature of the wave-front will be −5 dioptries. At the distance 0·333 metre the curvature will −3 dioptries, and so forth. The negative sign means that this wave has a divergent surface.

Now consider a positive lens having a convergivity or power of +12 dioptries. This means that it imprints on waves such a curvature that if the waves were coming straight, along parallel paths, it would cause them to converge to a distance of $\frac{1}{12}$ metre (or 0·083 metres) as measured forward to the principal focus on the right. Or, if the wave-front coming to the lens is already curved, the lens will imprint on the curved wave-front an additional curvature of +12 dioptries.

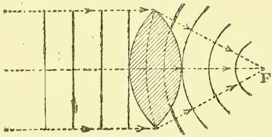

FIG. 6.

Now put together these two things. Let the light diverge from a point P at a distance u from the lens, and pass through the lens, so that it is brought to a conjugate focus at Q on the right; at what distance will this point be from the lens? The calculation is simple. For by the time the wave reaches the lens it has a curvature $\frac{1}{u}$, and the lens adds to this an additional im-

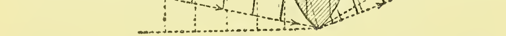

FIG. 7.

pressed curvature $\frac{1}{f}$; consequently the curvature that results must be simply $\frac{1}{u} + \frac{1}{f}$. Now if the distance from the lens to the point Q to which the waves are converged be called v, it is clear that $\frac{1}{v}$ must be the curvature of the wave as it emerges

from the lens, and which results from its initial and the impressed curvatures. Hence the well-known formula

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (2)$$

The formula (2) above may be read in words thus: the resultant curvature is equal to the algebraic sum of the initial curvature and the impressed curvature.

In interpreting this formula it must be remembered that distances reckoned back from the lens to the left are negative, the curvature being divergent. In the numerical example given above, where $u = -0 \cdot 200$, and $f = 0 \cdot 083$, we shall have—

$$\frac{1}{v} = -\frac{1}{0 \cdot 200} + \frac{1}{0 \cdot 083},$$

$$\frac{1}{v} = -5 + 12 \text{ (dioptries)},$$

or $$\frac{1}{v} = 7 \text{ dioptries},$$

whence $$v = \frac{1}{7} \text{ metre} = 0 \cdot 143 \text{ metre}.$$

In the optical trade there is used a very simple instrument, called a *lens-measurer*, for measuring the power of lenses. It is in reality a simple spherometer, so constructed that when it is pressed against the surface of a lens, the readings upon its dial are proportional to the amount of curvature of the surface. If it is pressed against the two surfaces successively, and the readings added, we obtain the total curvature. Now, if this were all one would—by using the formula (1), p. 36, need to multiply by the refractivity of the glass (i.e. by $\mu - 1$) in order to obtain the number of dioptries of its convergivity or power. But, to save trouble, the instrument makers have so constructed the scale that all the readings are already multiplied by $0 \cdot 51$ (the mean refractivity of crown glass); and what we really read off is the curvature already multiplied by this value. In fact, we read off the values of $(\mu - 1) \times \frac{1}{r_1}$ and $(\mu - 1) \times \frac{1}{r_2}$; and adding these together gives the total value $(\mu - 1) \times \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, which is the convergivity, or power, and is equal to $\frac{1}{f}$.

For convenience of reference, the next Tables, Nos. 29 and 30, give the corresponding values of Powers (Dioptries) and Focal Lengths both in metric units and in inches. It will be

FIG. 8.

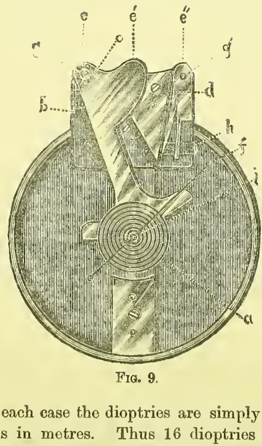

FIG. 9.

noted that in each case the dioptries are simply the reciprocals of the lengths in metres. Thus 16 dioptries corresponds to $\frac{1}{16}$ of a metre, or 0·0625 metre, or 62·5 millimetres.

29

POWER AND FOCAL LENGTH.

POWER or *Curvature.*	FOCAL LENGTH or *Radius of Curve.*		
Dioptries.	Metres.	Millimetres.	Inches.
0·25	4·0000	4000	157·48
0·5	2·0000	2000	78·74
0·75	1·3333	1333·3	52·49
1·00	1·0000	1000·0	39·37
1·25	·8000	800·0	31·50
1·5	·6667	666·7	26·24
1·75	·5714	571·4	22·50
2·00	·5000	500·0	19·68
2·25	·4444	444·4	17·50
2·5	·4000	400·0	15·75
2·75	·3636	363·6	14·32
3·00	·3333	333·3	13·12
3·5	·2857	285·7	11·25
4·0	·2500	250·0	9·84
4·5	·2222	222·2	8·75
5·0	·2000	200·0	7·87
5·5	·1818	181·8	7·16
6·0	·1667	166·7	6·56
7	·1429	142·9	5·62
8	·1250	125·0	4·92
9	·1111	111·1	4·37
10	·1000	100·0	3·94
11	·0909	90·9	3·58
12	·0833	83·3	3·28
13	·0769	76·9	3·03
14	·0714	71·4	2·81
15	·0667	66·7	2·63
16	·0625	62·5	2·46
17	·0588	58·8	2·32
18	·0555	55·5	2·19
19	·0526	52·6	2·07
20	·0500	50·0	1·97
25	·0400	40·0	1·58
30	·0333	33·3	1·31
40	·0250	25·0	·98
50	·0200	20·0	·79
60	·0167	16·7	·66
70	·0143	14·3	·562
80	·0125	12·5	·492
90	·0111	11·1	·437
100	·0100	10·0	·394
120	·0083	8·3	·328
140	·0071	7·1	·281
160	·0062	6·2	·246
180	·0055	5·5	·219
200	·0050	5·0	·197
250	·0040	4·0	·157
300	·0033	3·3	·131
400	·0025	2·5	·098
500	·0020	2·0	·079
1000	·0010	1·0	·039

30

FOCAL LENGTH AND POWER.

FOCAL LENGTH or *Radius of Curve.*			POWER. or *Curvature.*
Inches.	Millimetres.	Metres.	Dioptries.
1	25·4	0·0254	39·37
1·25	31·75	0·03175	31·50
1·5	38·10	0·03810	26·25
1·75	44·45	0·04445	22·50
2	50·80	0·05080	19·69
2·25	57·15	0·05715	17·50
2·5	63·50	0·06350	15·75
2·75	69·85	0·06985	14·32
3	76·20	0·07620	13·12
3·25	82·55	0·08255	12·11
3·5	88·90	0·08890	11·25
3·75	95·25	0·09525	10·50
4	101·6	0·10160	9·84
4·5	114·3	0·11430	8·75
5	127·0	0·12700	7·87
5·5	139·7	0·13970	7·16
6	152·4	0·15240	6·56
6·5	165·1	0·16510	6·06
7	177·8	0·17780	5·62
8	203·2	0·20320	4·92
9	228·6	0·22860	4·37
10	254·0	0·25400	3·94
11	279·4	0·27940	3·58
12	304·8	0·30480	3·28
13	330·2	0·33020	3·03
14	355·6	0·35560	2·81
15	381·0	0·38100	2·62
16	406·4	0·40640	2·46
17	431·8	0·43180	2·32
18	457·2	0·45720	2·19
20	508·0	0·50800	1·97
22	558·8	0·55880	1·79
24	609·6	0·60960	1·64
26	660·4	0·66040	1·51
30	762·0	0·76200	1·31
35	889·0	0·88900	1·12
40	1016·0	1·01600	0·984
48	1219·2	1·21920	0·820
60	1524·0	1·52400	0·656
72	1828·8	1·82800	0·547
100	2540·0	2·54000	0·394

(I.) *To convert Dioptries to Metres.*—Divide 1 by the number of Dioptries.

(IA.) *To convert Metres to Dioptries.*—Divide 1 by the number of Metres.

(II.) *To convert Dioptries to Millimetres.*—Divide 1000 by the number of Dioptries.

(IIA.) *To convert Millimetres to Dioptries.*—Divide 1000 by the number of Millimetres.

(III.) *To convert Dioptries to Inches.*—Divide 40 by the number of Dioptries.

(IIIA.) *To convert Inches to Dioptries.*—Divide 40 by the number of Inches.

N.B.—The exact number of inches in the metre is not 40, but 39·3708; but the simple number 40 is sufficiently near in calculations about spectacle lenses.

31 Effect, on Apparent Power of a Lens, of Distance from place where its effect is to be produced.

It follows from the first principles of propagation of waves that the curvature of any wave-front *must* alter as it travels onwards. An expanding wave (Fig. 10) as it diverges from a point F becomes less curved. On the other hand, a converging

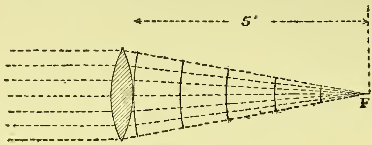

FIG. 10.

wave-front becomes more highly curved as it nears its focus at F (Fig. 11). Suppose a beam from a distant object to have passed through a lens of +8 dioptries. The wave-front which emerges will have a curvature (convergent) of + 8 dioptries printed upon it, and would converge to a focal point about

5 inches beyond the lens. But this convergent wave is growing more convergent, and by the time it has passed *one inch* further along, the convergency is as great as if a lens of about $+10$ **D** (i.e. of 4 inches focal length) had been used. Let the true

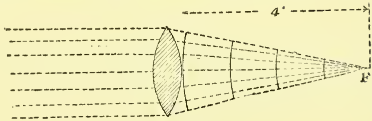

FIG. 11.

power of a lens, expressed in dioptries, be called p, and its apparent power at a distance d (in terms of the metre) further along be called p', then p may be calculated by the formula

$$p' = p\,\frac{1}{1 - p\,d}.$$

If d is given in inches, the rule becomes

$$p' = p\,\frac{40}{40 - p\,d};$$

or, if d is given in millimetres,

$$p' = p\,\frac{1000}{1000 - p\,d}.$$

As an example: the apparent power of a $+6\cdot5$ **D** lens at a point 40 millimetres further on

$$= 6\cdot5 \times \frac{1000}{1000 - 6\cdot5 \times 40} = 6\cdot5 \times \frac{1000}{1000 - 260} = \frac{6500}{740} = 8\cdot7\ \mathbf{D}.$$

As another example: the apparent power of a -10 **D** lens at a point 2 inches further on

$$= -\mathbf{10} \times \frac{40}{40 + 10 \times 2} = -10 \times \frac{40}{40 + 20} = -\frac{400}{60} = -6\cdot6\ \mathbf{D}.$$

One practical result of this effect is that a person having presbyopia or hypermetropia, and using + lenses, can increase their apparent power, when desiring to read a book, by pulling them down lower upon his nose, and can diminish their apparent power when desiring to view distant objects, by pushing them up close to his eyes.

It is possible to *neutralise* any lens of low power by combining it with one of equal and opposite power; thus, if a + 4 **D** lens is held in contact with a − 4 **D** lens, the magnifying power of the + lens is neutralised by the minifying action of the − lens. The reason that complete neutralisation can be thus effected is because when we are dealing with two *thin* lenses that are in contact and centred on the same axis, the power of the two jointly is equal to the sum (algebraic) of their individual powers.

Neutralisation, therefore, affords an excellent way of ascertaining the power of a lens of unknown denomination. If the observer is furnished with a *trial-case* of lenses, both positives and negatives, in a graduated series from 0·25 **D** to 20 **D**, he will be able to test the power of any spherical lens by simply combining it with another spherical lens, and trying various lenses until he succeeds in neutralising it. It is easy to ascertain neutralisation by the following method :—

Hold the lens about a foot from the eye, and while looking through it any object—best of all at horizontal objects such as window-bars or lines of large point—move it up and down through a distance of half an inch or so. If the lens is a + lens (magnifying lens), then when the lens is moved *up*, the object seen through it will seem to move *down*; and when the lens is moved *down* the object will appear to move *up*. In other words, in the case of a + lens *the object appears to move* **against** *the lens.* On the other hand, if the lens is a − (concave or minifying) lens, then when the lens is moved *up*, the object appears to move *up*; or, in other words, in the case of a − lens *the object appears to move* **with** *the lens.* If the lens is very slightly +, or very slightly −, the motion will be slight; and with a piece of flat glass there will be no motion at all. Hence, in neutralising a lens, one tries it first with one lens and then with another, watching whether, on looking through the combination, the apparent motion of an object is 'with' or 'against.' When perfectly neutralised, there will be no motion.

When, however, *thick* lenses are in question, it is only possible to neutralise perfectly a + lens with an equal − lens, if their forms are such as to fit together and give two flat outer

surfaces. For example, let a thick plano-convex + 20 **D** be combined with a thick plano-concave − 20 **D**. If their curved faces are fitted together they will completely neutralise one another; whereas, if they are combined with their flat faces together, so that one outer face is convex and the other equally concave, they will not precisely neutralise one another, but will be slightly positive. In fact, if a lens is made with one face with a curvature $+\frac{1}{r}$, and the other with an equal but opposite curvature $-\frac{1}{r}$, of a thickness t, and of a glass having a refractive index of 1·5, it will have a power of $t \div 6\,r^2$ dioptries (the values of r and t being given in metres). If r and t are given in millimetres, the power of this lens will be $\div\, 0{\cdot}006\, r^2$. The reason why the effect of the convex surface on one side is not completely neutralised by the equal concave surface on the other side is that in passing through the thickness of the lens the curvature of the wave-surface is altered [see **31**] as it converges or diverges on its way.

33 TRANSPOSITIONS OF SPHERICAL LENSES.

As stated in **16**, the convergivity of a lens depends on the refractivity of the glass and on the total curvatures of its two surfaces. In fact, formula 1 on page 36,

$$\frac{1}{f} = (\mu - 1) \times \left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

may be written:

Convergivity = refractivity × sum of the curvatures.

Now the sum of the curvatures might be made up in many different ways to the same value. A bi-convex lens having the radii of curvature of its faces both 600 millimetres, will have total curvature $= \frac{1}{0{\cdot}600} + \frac{1}{0{\cdot}600} = \frac{1000}{600} + \frac{1000}{600} = \frac{2000}{600}$ $= 3{\cdot}33$ dioptries; and if the glass had refractivity $= 0{\cdot}52$, the power of the lens would be $0{\cdot}52 \times 3{\cdot}33 = 1{\cdot}7$ dioptries. But an exactly equal power would be attained by making the

lens plano-convex, provided the amount of the curvature on the bulging face were exactly double that of either of the faces of the bi-convex lens. In other words the radius of curvature of the plano-convex must be half as great, viz. 300 millimetres. In that case the total curvature will be $\left(0 + \frac{1}{0\cdot300}\right)$ $= \frac{1000}{300} = 3\cdot33$ dioptries as before.

In choosing spectacles some persons prefer a particular shape of lens, which has the advantage that it leaves more room for the eye-lashes, namely, the kind which is hollow on the surface toward the eye, while bulging on the outer surface. Such a lens is sometimes described as *periscopic.* Another name for such a lens is a *meniscus.* It would be described as a *convex meniscus* or a *positive meniscus,* if the convex or positive curvature of the outer surface were stronger than the concave or negative curvature of the inner surface; or as a *concave meniscus* or *negative meniscus* if the inner negative curvature were the stronger. The term "periscopic" is usually applied to positive meniscus lenses only. If the curvatures of the faces of a periscopic lens are chosen so that the total curvature is the same as that of any given lens, it will have the same power. For example, if a periscopic lens were chosen with curvatures of **+ 5** dioptries on the outer face, and of **− 1·67** dioptries on the inner face, the algebraic sum of these surface curvatures would be 3·33 as before.

By using a "lens-measurer" (see p. 39) to measure the curvatures of the faces, it is very easy to verify the rule that the power depends on the total curvature, negative curvatures being of course subtracted.

Any optician who is asked to furnish, say, a + 4 **D** lens, can furnish either a plano-convex, with all the curvature on one face, or a bi-convex with half the total curvature on each face, or a periscopic lens, with more than + 4 **D** on its front face, and a negative curvature on the inner face. The calculation for thus finding new curvatures which will produce the same optical effect is called "transposition." In transposing simple spherical lenses, there is but one simple rule to remember, namely, that if you increase the curvature of one face by any particular amount, you must decrease the curvature of the other face by a precisely equal amount.

Example.—What radius of curvature must be given to the front surface of a periscopic lens, the back surface having a radius of 3 cms., the power being 5 D and the index of the glass 1·5? Notice that the lens being periscopic, the back curvature and radius are negative.

$$\text{The algebraic sum of the curvatures} = \frac{5}{1\cdot 5 - 1} = 10.$$

$$\text{Curvature of the back surface (in the same units)} = -\frac{\frac{1}{3}}{100} = -33 \;.$$

Hence we have—

$$\text{Front curvature} + (-33\cdot 3) = 10;$$

that is,

$$\text{Front curvature} = 43\cdot 3.$$

$$\text{The radius of the front surface} = \frac{1}{43\cdot 3}\ \text{metres} = \frac{100}{43\cdot 3} = 2\cdot 3\ \text{cms.}$$

34 CARDINAL POINTS OF LENSES.

In order to treat the problems of thick lenses and systems of lenses, which would otherwise be very complicated, the German geometrician Gauss introduced the method of representing a lens system by a set of points and planes. A modification of that method is here briefly summarised. Though in

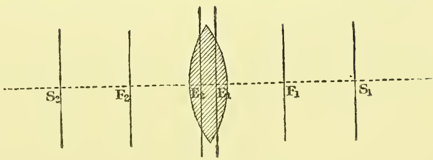

Fig. 12.

the accompanying figure a simple bi-convex lens is shown, a similar system of points and planes can—with certain modifications—be found for any lens or system of lenses.

These points and plains are in pairs as follows:—

First Pair of Points.—E_1 E_2 called the "Equivalent" points, or "Principal" points, or "Optical Centres" of the lens. Their property is that any ray which moves towards one of

them will, after traversing the lens, emerge parallel to its former path, but as if it had passed through the second of them, as in Fig. 13.

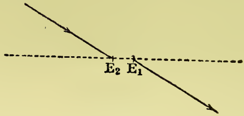

FIG. 13.

Second Pair of Points.—F_1 F_2 called the principal *Foci.* Light passing through the lens parallel to its chief axis will be (if the lens is positive) conveyed to one of these two principal foci, according to its direction. The distance from either of the principal foci, measured back to the corresponding principal point, as from F_1 to E_1 (Fig. 12) is the *true focal length of the lens*, or *the equivalent focal length of the lens.*

Third Pair of Points.—S_1 S_2 called the "Symmetric" points. These are situated on the principal axis at double the focal distance from the principal points. They are images of one another: that is, an object placed at S_1 will have its image at S_2, and *vice versâ.*

Fourth Pair of Points.—In the rare cases in which the two surfaces of a lens are in contact with two different media—as for example, a lens with air in front and water behind, the optical centres of the system are no longer at E_1 E_2, but are displaced along the axis toward the denser of the two surrounding media. They are called the "Nodal Points." Thus in the eye itself the nodal points lie further back than the two equivalent points of the crystalline lens. In such cases the property of acting (as described above) as an optical centre is transferred from the equivalent points to the nodal points.

Fifth Pair of Points.—Abbe has shown that the field of vision of every optical system is bounded by what he terms the *Pupils* of the system—the *Entrance-pupil* on one side, and the *Exit-pupil* on the other. These are situated at two points conjugate to one another. See Art. **63**.

The pairs of planes, all being drawn perpendicular to the axis, are :—

1. *Equivalent Planes* or *Principal Planes*, drawn through the

two equivalent points. Their property is that any ray of light which meets any point in one of these planes emerges from the lens as though it had been transferred straight across from one plane to the other parallel to the axis, as illustrated in Fig. 14.

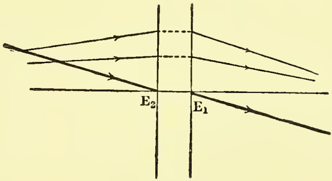

FIG. 14.

2. *Focal Planes*, drawn through the two principal foci. Their property is that light starting from any point (such as P) (Fig. 15) in one of these planes will, after passing through the lens, emerge in a parallel beam, parallel to an oblique or secondary axis through that point.

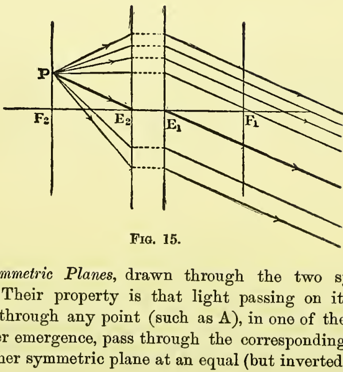

FIG. 15.

3. *Symmetric Planes*, drawn through the two symmetric points. Their property is that light passing on its way to the lens through any point (such as A), in one of these planes will, after emergence, pass through the corresponding point A′ in the other symmetric plane at an equal (but inverted) distance

sideways from the axis. (In Fig. 16, to find where the ray A B goes to, make y_1 downwards equal to y_2 upwards.)

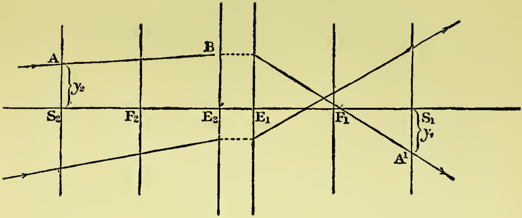

Fig. 16.

35 Positions of the Equivalent Points, or Principal Points, and Planes.

In simple equi-convex lenses the two equivalent planes and points are situated symmetrically at a distance apart approximately equal to one-third the thickness of the lens. But in those simple convex lenses that are unequally curved on the two faces they are displaced toward the face of greater curvature. In convex meniscus lenses of deep curvature, they may even be displaced altogether outside the lens. See Fig. 17.

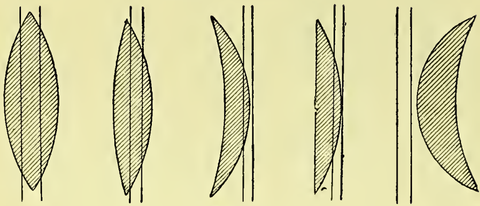

Fig. 17.

A similar series of sketches for negative lenses is given on next page (Fig. 18).

Positions of the Equivalent Points—*continued.*

The formulæ for finding the positions of the two equivalent points are complicated, but the distance between them (or the "equivalent thickness") is, with approximate accuracy, expressed as follows :—

$\Delta = t\dfrac{\mu - 1}{\mu}$; where Δ is the distance between the two equivalent points, t the thickness of the lens at its middle, and μ the index of refraction of its material. For crown glass ($\mu = 1 \cdot 5$) it follows that Δ is approximately one-third of t.

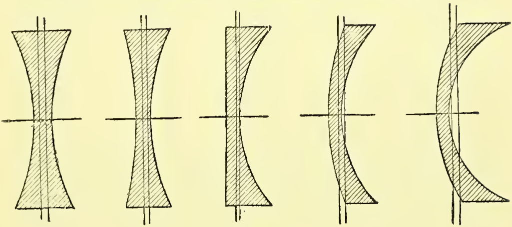

FIG. 18.

36 Formulæ connecting the Cardinal Points with the Form of the Lens.

The positions of the cardinal points in a thick lens can be calculated when the radii of curvature of its surfaces, and the refractive index (μ) of the glass are known. If r_1 is the radius of the first surface, and r_2 that of the second surface of the lens, and O_1 be the front vertex of the lens, and O_2 the back vertex and the thickness from O_1 to O_2 be called t, then the distance of E_1 from O_1 measured inwards is

$$\frac{r_1 t}{\mu(r_1 + r_2 - t) + t};$$

and the distance of E_2 from O_2 measured inwards is

$$\frac{r_2 t}{\mu(r_1 + r_2 - t) + t}.$$

The width between E_1 and E_2, or "equivalent thickness"

$$\Delta = \frac{t(r_1 + r_2 - t)(\mu - 1)}{\mu(r_1 + r_2 - t) + t}.$$

When t is small compared with $r_1 + r_2$ this reduces to

$$t\frac{(\mu - 1)}{\mu},$$

as given above.

Or in the case of lenses of crown glass, for which $\mu = 1 \cdot 5$, the width between E_1 and E_2 is approximately one-third of t.

The true focal length or "equivalent focal length" $F_1 E_1$ or $E_2 F_2$ is found by the formula

$$f = \frac{\mu}{\mu - 1} \cdot \left\{\frac{r_1 r_2}{\mu(r_1 + r_2 - t) + t}\right\}.$$

These formulæ are the same for all lenses, both positive and negative, including meniscus lenses, provided it be remembered that the radii of curvature of convex surfaces are reckoned positive, and those of concave surfaces are reckoned negative, whichever way they face.

37 LENS COMBINATIONS.

If two lenses are placed some distance apart, it is known that the combination is "equivalent" to a single lens differing in thickness and power from either of them. That is to say that so far as focal lengths, &c., are concerned, it is possible to find a single lens which shall have the same power as the combination. But the single "equivalent" lens would possess defects of spherical and chromatic aberrations, the avoidance of which is one of the objects to be attained by combining lenses together.

Two Thin Lenses in Contact.—In this case the resultant power is simply the sum of the two. Thus a lens of + 3 **D** combined with one of + 2·25 **D**, gives it equivalent to a single lens of + 5·25 **D**. Or a lens of + 6 **D**, combined with one of − 3·5 **D**, gives a single lens of + 2·5 **D**.

Two Thin Lenses at a Distance Apart.—The rule is that the

equivalent lens will have a power equal to that which the two would have when close together, less an amount equal to the product got by multiplying together the powers of the two lenses and the distance between them expressed as in decimals of a metre.

Or, in symbols, the resultant power is $= P_1 + P_2 - P_1 P_2 w$; where P_1 and P_2 are the powers of the two lenses, and w the width or distance between them. Example: let the two lenses be $+ 3$ **D** and $+ 5$ **D**; and let $w = 25$ millimetres $= \cdot 025$ metres. Then the power of the equivalent lens will be $= 3 + 5 - (15 \times \cdot 025) = 8 - \cdot 375 = 7 \cdot 625$ dioptries.

As another example, two lenses of $+ 12$ **D** each, set at 20 millimetres apart, will be equivalent to $12 + 12 - (12 \times 12 \times \cdot 02) = 24 - 2 \cdot 88 = 21 \cdot 12$ dioptries.

As a third example, two lenses of $+ 12$ **D** and $- 8$ **D** at 6 millimetres apart will be equivalent to $12 - 8 - (12 \times - 8 \times \cdot 006) = 4 + \cdot 576 = 4 \cdot 576$ dioptries.

A Huygenian eye-piece with lenses having respectively $+ 15$ **D** and $+ 45$ **D** at 40 millimetres apart, will be equivalent to $15 + 45 - (15 \times 45 \times \cdot 04) = 60 - 27 = 33$ dioptries.

38 CARDINAL POINTS OF COMBINATIONS OF LENSES.

In many optical instruments, two or more spherical lenses are placed in position, properly centred on the same axis, and at a distance from one another. In such cases the question often arises: what single lens—if any—would be equivalent to the system? This *equivalent lens* could not in all cases be constructed, even if all aberrations were absent. If the positions of the cardinal points of each separate lens are known, and also the distance between them, it is easy by geometry to determine the positions of the cardinal points of the system as a whole; and if certain relations connecting the path of the light through a simple lens with the positions of its cardinal points are remembered, the application to the case of any given combination of lenses becomes quite simple.

In the case of a single lens, suppose F_1 and F_2 are the principal foci, E_1 and E_2 the equivalent points. Let R L be the direction of a given ray. It is required to find its direction after passing through the lens. Draw OE_2, $E_1 O'$, as a secondary axis parallel to R L. It meets the focal plane in N. Then, by the definition of a focal plane, it follows that any plane

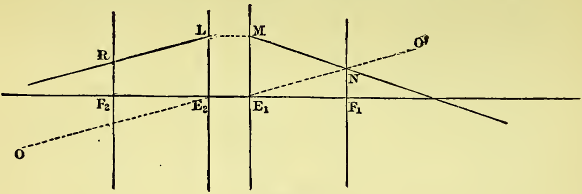

Fig. 19.

wave travelling along $O E_2$ will come to a focus at N. Therefore that part of the wave which travels along R L will also come to the focus at N. Therefore draw R L until it meets at L the equivalent plane; transfer it across to M, and join M N. The line M N is the direction in which the ray R L will be directed after passing through the lens. Clearly this,

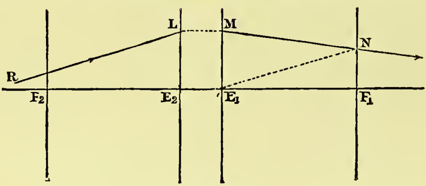

Fig. 20.

point N might have been found, without drawing the part $O E_2$ by merely drawing E_2O' parallel to R L. Hence we get the following universal construction. If a ray intersects the first of the equivalent planes which it meets at L, draw from the second of the equivalent points a line parallel to

the ray until it meets the focal plane at a point (here called N). Then having transferred the ray across the equivalent thickness from L to M, join M to N, thus finding the path taken by the ray.

By using this construction in succession for each lens of a

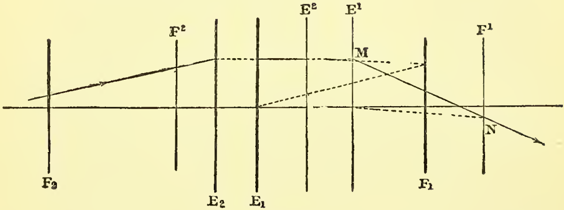

Fig. 21.

combination, it is possible to show how any incident ray will emerge from the combination.

Take first the case of two positive lenses placed apart at any distance less than the focal length of either of them.

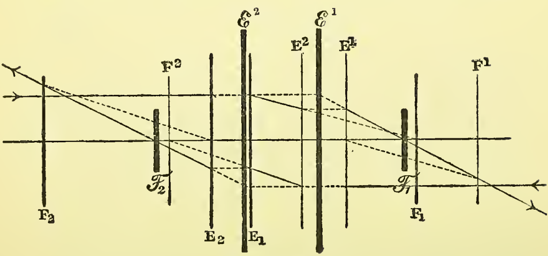

Fig. 22.

Fig. 21 gives the construction of the path of the ray as it emerges. being shown at M N. In this figure we have used the symbols

E_2E_1 for the equivalent points of the first lens; and E^2E^1 with the numerals affixed to the top, for those of the second lens.

In order to find the cardinal points of the system as a whole, we take rays parallel to the principal axis, since these, after refraction, cross the axis at the principal focus of the system, as in the case of a single lens. Then to find the position of the equivalent planes of the system, the lines representing the initial and final directions of the ray must be produced till they meet, giving where they intersect the positions of these two equivalent planes E_2 E_1; which are of course represented (as in Fig. 22) by lines perpendicular to the axis. The reason of this will be made clear by considering the position of the equivalent planes in a simple lens with respect to the initial and final paths of a ray originally parallel to the axis.

Let us now take one other case, namely that in which there are two convergent lenses at a distance apart greater than the sum of their focal length. Let the focal length of the first lens be f, and that of the second f^1; f being taken as greater than f^1.

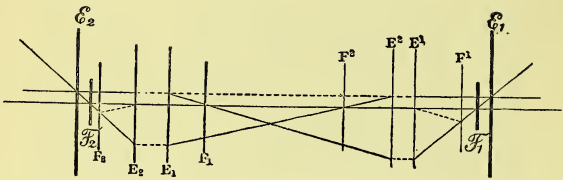

Fig. 23.

In this case, though F_1 and F_2 are both real foci, the system acts like a divergent lens; the rays (when only their external parts are considered) *apparently* being rendered divergent without crossing the axis. It should also be noted that the true focal length F_1 E_1 of the system is *negative*, the equivalent planes lying outside the principal foci.

The following table gives the character of the equivalent foci and of the focal lengths in a number of cases, where the distance between the two lenses (*i.e.* the distance between their

adjacent equivalent planes) is varied, beginning with the lenses in contact:—

Case.	Distance apart.	Parallel Rays, from Lens No. 1 to Lens No. 2.		Parallel Rays, from Lens No. 2 to Lens No. 1.		Kind of System which results.
		Nature of Focus.	Equivalent Focal Length.	Nature of Focus.	Equivalent Focal Length.	
1	$= 0$	real	$\frac{f_1 f_2}{f_1 + f_2}$	real	$\frac{f_1 f_2}{f_1 + f_2}$	convergent
2	$< f_2 > 0$	real	positive	real	positive	"
3	$= f_2$	real	positive	real, and on centre of No. 1	$+ f_2$	"
4	$< f_1 > f_2$	real	positive	virtual	positive	"
5	$= f_1$	real, and on centre of No. 2	$+ f_1$	virtual	positive	"
6	$< f_1 + f_2 > f_1$	virtual	positive	virtual	positive	"
7	$= f_1 + f_2$	none	∞	none	∞	afocal
8	$< \infty > f_1 + f_2$	real	negative	real	negative	divergent
9	$= \infty$	none	none	none	none	none

In diverging lenses, the focal lengths being negative, the foci are on opposite sides of the lens with respect to equivalent points.

Thus for a converging lens, F_1 is on the same side as E_1, as in Fig. 24. The true focal length is $E_1 F_1$. For a diverging

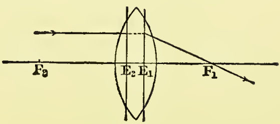

Fig. 24.

lens F_1 is on the opposite side of the lens from E_1; and the true focal length is still $E_1 F_1$, and being reckoned backwards from E_1 is negative.

Particular Cases.

Several particular cases arise that are of interest in connexion with instruments.

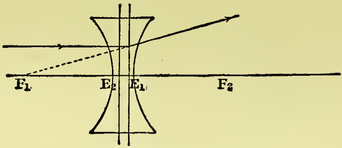

Fig. 25.

Case I. *Two + lenses of equal focal length, at a distance apart equal to double the focal length.*—This combination is afocal, it does not magnify or minify, but merely inverts the image. Such a combination can be used as an *erector* in the tube of a telescope or microscope to re-erect an otherwise inverted image.

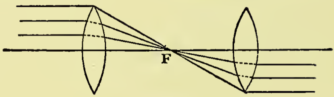

Fig. 26.

Case II. *Two + lenses of unequal focal lengths, at a distance apart equal to the sum of the focal lengths.*—This combination is also afocal; that is to say parallel rays entering it emerge as parallel, and rays that are nearly parallel emerge as nearly parallel. An

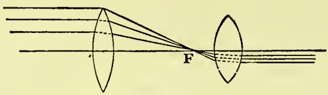

Fig. 27.

eye, if focussed for distant objects, will through this combination see distant objects in focus, magnified if the shorter focus lens is nearer the eye, minified if the longer focus lens is the nearer. In each case the image will be inverted. This is in fact the

case of the simple *astronomical telescope*, but, as habitually used for normal eyes (for which it is convenient that the light that enters the eye should diverge as from the *punctum proximum* or near point), the eye-piece lens is pushed a little nearer toward the object-glass, thus reducing the distance between the lenses to a little *less* than the sum of the two focal lengths.

Case III. *Two + lenses of unequal focal lengths, at a distance apart equal to the difference of their focal lengths.*—This is a combination which exists in the case of Huygenian eye-pieces.

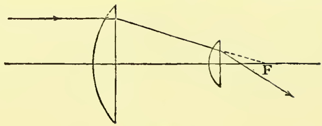

Fig. 28.

Case IV. *A + lens of long focus, and a – lens of short focus at a distance apart equal to the difference of their focal lengths* (*i.e.* equal to their algebraic sum).—This combination is afocal, having similar properties to No. II, except that images are not inverted. It is in fact, the case of the Galilean telescope. The opera-glass, reduced to its simplest elements, and adjusted for an eye focussed for distant objects, is the same combination.

39 COMBINATIONS OF TWO THICK LENSES.

In this section the following symbols are used :—

f_1 f_2, the true *focal lengths* of the two lenses respectively.

p_1 p_2, the *powers* (in *dioptries*) of the two lenses respectively.

t_1 t_2, the *equivalent thicknesses* of the two lenses (*i.e.* the respective distances between the equivalent planes of each).

c, the distance (in metres) between their adjacent planes, or true distance apart between the lenses.

F, the *resultant focal length* of the combination.

P, the *resultant power* (dioptries) of the combination.

d, the *resultant equivalent thickness*, or distance apart of the two resultant equivalent planes of the system.

The following formulæ then hold good:—

$$\mathrm{F} = \frac{f_1 f_2}{f_1 + f_2 - c} = \frac{1}{\mathrm{P}} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mathrm{P} = p_1 + p_2 - p_1 p_2 c = \frac{1}{\mathrm{F}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$d = t_1 + t_2 - \frac{c^2 p_1 p_2}{\mathrm{P}} = t_1 + t_2 - \frac{c^2 \mathrm{F}}{f_1 f_2}. \quad . \quad (3)$$

Further, the distances of the resultant equivalent planes, measured *inwards*, from the outer equivalent planes of the component lenses f_1 and f_2 are respectively:—

$$\frac{\mathrm{F}c}{f_2} = \frac{c p_2}{\mathrm{P}}, \quad \text{and} \quad \frac{\mathrm{F}c}{f_1} = \frac{c p_1}{\mathrm{P}}.$$

In applying these formulæ—*and they are applicable to any combination* of two lenses—it must be remembered that divergent lenses have negative focal lengths, and that distances with negative signs must be reckoned outwards.

Examples.—(1) Find the equivalent values for a system of two lenses of which the values are as follows:—$f_1 = +4$ inches, $f_2 = +3$ inches; $t_1 = 0{\cdot}15$ inch; $t_2 = 0{\cdot}2$ inch; $c = 1{\cdot}5$ inch. *Answer.* $\mathrm{F} = 2{\cdot}18$ inch.; $d = -0{\cdot}059$ inch.

(2) Find the equivalent values for a combination of two lenses having following values:—$p_1 = 20$ dioptries, $p_2 = 8$ dioptries; $t_1 = {\cdot}003$ metres, $t_2 = 0{\cdot}001$ metre; $c = 0{\cdot}020$ metre. (N.B.—For dioptric calculations all length measurements should be converted to decimals of the metre.) *Answer.* $\mathrm{P} = 24{\cdot}4$ dioptries; $d = 0{\cdot}00138$ metre or $1{\cdot}38$ millimetre.

It will be noticed, as an obvious deduction from formulæ (1) and (2) above, that if both the lenses are positive, increasing the distance (c) between them always increases the equivalent focal length, and reduces the equivalent power. It also follows from formula (2) that if one of the two component lenses is negative, so that the term $-p_1 p_2 c$ has a positive value, any increase of c will increase the power or shorten the equivalent focal length. It is also obvious from (3) that the equivalent thickness of a combination of two positive lenses is reduced by separating them from one another, and that if c is made sufficiently great the resultant equivalent thickness will become zero, or if made still greater may even have a negative value, the two equivalent planes crossing one another. In the case of many camera lenses

the distance to which the two components are separated is often so great as to make the equivalent lens of no thickness, or even of a negative thickness. An approximate formula for finding the distance between the components which will make the equivalent thickness zero is—

$$c = \sqrt{(t_1 + t_2)(f_1 + f_2)} - \tfrac{1}{2}(t_1 + t_2) \quad . \quad . \quad (4)$$

Example.—Two lenses have equivalent thicknesses of 6 mm. and 9 mm. respectively; and their focal lengths are + 200 mm. and + 100 mm. The distance c at which they must be placed apart so as to reduce the equivalent thickness to zero is, by this formula, 36·2 mm.; the equivalent focal length of the combination then being = + 75·8 mm.

40 CYLINDRICAL LENSES.

Cylindrical lenses may be described as lenses having one (or both) of their surfaces shaped as a portion of a cylinder, as ordinary lenses are of a sphere.

Fig. 29 represents, in perspective, four cylindrical lenses before their corners have been ground away to fit them into

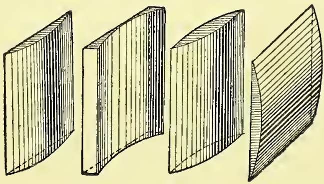

Fig. 29.

frames. The first one (on the left) is a simple plano-cylindrical convex (or positive) lens. The second is a plano-concave (or negative) cylinder. The third is a bi-convex cylinder. The fourth is a plano-convex cylinder, in fact the same as the first, except that it has been turned round. Nos. 1, 2, and 3 of this figure would be described as having their axes vertical, while

No. 4 has been turned so that its axis is horizontal. This term is illustrated in Fig. 30, from which it will be seen that direction (called the *axis* of a cylindrical lens) is the same as that of the axis of the imaginary cylinder, of which it forms a part. *The optical effects of a cylindrical lens depend not only on the curvature of its surfaces and on the refractivity of the glass, but also on the direction in which its axis is set.*

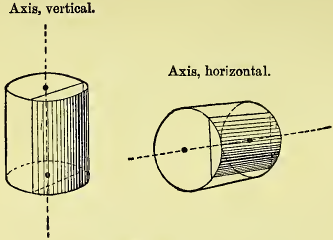

Fig. 30.

A lens which is cylindrical on one side only and flat on the other, is called a plano-cylinder. If cylindrical on both sides and with cylindricity that acts along the same axis its effect is still simply cylindrical, and it is still called a *simple cylindrical lens*, or, briefly, a *simple cylinder*.

Often, however, lenses are ground with a cylindrical curvature (positive or negative, i.e. bulging or hollow) on one face, while the other face, instead of being left flat, is ground with a spherical surface (bulging or hollow) like the face of an ordinary lens. Such lenses are called *sphero-cylindrical lenses*, or, briefly, *sphero-cylinders*.

Sometimes, but not often, lenses are ground with a cylindrical surface on each face, the axes on the curvatures on the two faces being set at right angles to each other—if one axis is vertical, the other will be horizontal. Such lenses are described as *crossed cylinders*.

The optical property of a simple cylindrical lens is, that its power of conveying or diverging the waves of light is exercised unequally in different meridians. For example, if a cylindrical lens is set, as in Fig. 31, with its axis vertical, being equally

thick from top to bottom, it will produce no refraction in the vertical meridian, and will not cause the light that passes through the top part to meet the light that passes through the bottom part. On the other hand, it will refract the light from right and from left, converging these rays toward the middle, and therefore causing the rays of a parallel beam to meet and

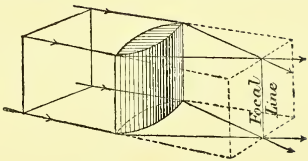

Fig. 31.

cross not at a *point* but at a *line.* In brief, a + cylinder lens produces a *focal line.* If the lens were turned about so that its axis is set horizontal, as in Fig. 32, then it will have no refraction in the horizontal meridian, but will refract in the vertical meridian, producing a *focal line* that is horizontal also. A parallel beam passing through a simple cylindrical lens produces a *focal line* at its principal focus, the line being parallel to the axis of the lens.

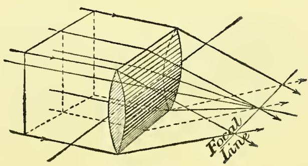

Fig. 32.

If experiments are made by covering up a cylindrical lens with an opaque screen in which a narrow slit has been cut, it will be found that if the screen is laid on the lens so that the slit is parallel to the axis, the lens produces *no* refraction—it

acts merely like a bit of *flat* glass. If the slit is turned so that it lies across the axis, then the refraction is found to be as great as possible. If the slit is set at any intermediate refraction the apparent refracting power of the lens will be of intermediate value. The usual way of stating this is:—*the refracting power of a cylindrical lens is a maximum in a meridian at right angles to the axis*, and is *zero in the meridian of the axis;* while it has intermediate values at intermediate angles. Thus, for example, a + 6 **D** cylindrical lens will have power + 6 **D** in the meridian at right angles to its axis. It will have no power in the meridian parallel to the axis. It will have a power of + 3 **D** in a meridian at 45° to the axis.

Similarly in the case of negative or diverging cylindrical lenses, such as Fig. 33. If set with axis vertical they diverge

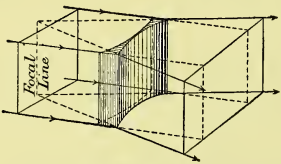

Fig. 33.

the light in the horizontal direction only, having no refractive power in the vertical meridan. In this case there is produced a virtual focal line, the light diverging as if from a line of light behind the lens.

41 SPHERO-CYLINDRICAL LENSES.

The effect of a cylindrical lens upon a converging pencil of light is to be noted. Suppose a beam of parallel light were to fall upon an ordinary converging lens (Fig. 34) of such power that it would thereby be converged to a point F_2 as the principal focus. If now a + cylindrical lens be interposed somewhere between the lens and F_2, with its axis vertical, seeing that

it has its refracting power greatest in the horizontal meridian, it will bring the light to a focus at a line at some spot, F_1, nearer in than F_2. But as this cylindrical lens has no power in the

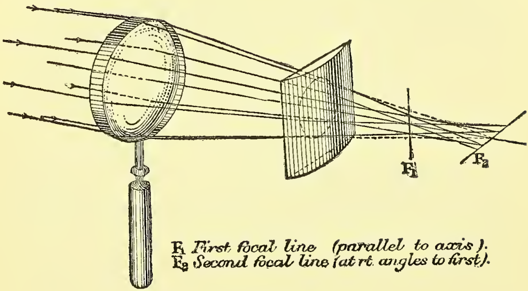

FIG. 34.

vertical meridian it cannot prevent all the top and bottom portions of the light from being converged by the lens to the middle level at F_2, where therefore there will appear a horizontal focal line.

Combination of Spherical and Cylindrical Lenses.—The effect just noted would still occur if the spherical lens and the cylindrical lens were put close together in contact. It would also occur if a lens were ground with a + spherical curvature on one face, and a + cylindrical curvature on the other face. In fact such a lens—a "sphero-cyl"—would produce on a parallel beam of light the effect of giving *two foci, both of them lines.* There would be a first focal line parallel to the axis of the cylinder, at a distance corresponding to the sum of the powers of the two lenses; and a second focal line, at right-angles to the first, at a distance corresponding to the power of the cylinder alone.

For example, if a + 4 **D** sph. were compounded with a + 4 **D** cyl. with its axis vertical, there would be a first focal line (vertical) at a distance of $\frac{40}{8} = 5$ inches (corresponding to

a power of 8 **D**) and a second focal line (horizontal) at a distance of $\frac{40}{4}$ =10 inches (corresponding to the power of 4 **D**).

To ascertain the Axis of a Cylindrical Lens.—The simplest method is to apply the "lens-measurer" (Art. **28**), turning it round on the cylindrical surface until it reaches the position indicating zero curvature.

Another method is to look through the lens at some fixed line or mark, and shift the lens rapidly to and fro, as in "neutralising" (Art. **32**). Whether the cyl. lens be + or –, no movement of the object is seen if the line along which the lens is moved is parallel to its own axis. For sphero-cylindrical lenses first neutralise the spherical part, and then test for axis as above.

42 PROPERTIES OF CROSSED CYLINDERS.

In dealing with this branch of our subject, we must glance back at the results previously arrived at in connection with simple spherical lenses of low power, namely, that the combined power of two or more lenses on the same axis and close together, is equal to the sum of their respective powers, and the converse that any given lens may be replaced by two or more on the same axis and close together, the sum of whose powers is equal to that of the given lens.

This clearly will apply also to simple cylinders *when their axes of figure are parallel*, for example two cylinders with their axes both vertical, or two cylinders with their axes both horizontal.

Thus we may replace a + 5 **D** cylinder with a + 3 **D** cylinder and a + 2 **D** cylinder together, or with a + 8 **D** cylinder and a – 3**D** cylinder together, or in fact with any two, or more, the sum of whose powers is equal to + 5 **D**.

Now suppose we have two crossed cylinders of equal power (as in Fig. 35): they will act like a sphere; for one will produce a refraction in one meridian, and the other an equal refraction in the meridian at right angles. For example, if a + 3 **D** cyl. be set with axis at 15°, and another + 3 **D** cyl. be set exactly across it (i.e. axis at 105°), they will together act simply

like an ordinary + 3 **D** sphere having a point focus. In the same way two crossed equal negative cylinders act merely as a negative sphere as is shown in Fig. 36. If however the crossed cylinders are not of equal power we may replace the first with two others, one having a power equal to that of the second, and

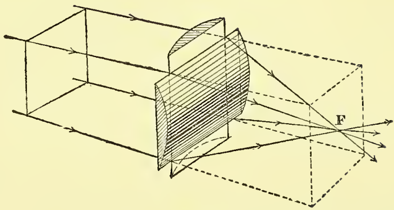

Fig. 35.

the other having a power equal to the difference between those of the first and second. We are thus left with (1) two crossed cylinders each of power equal to that of the second, and (2) a cylinder of power equal to the difference between those of the first and second, and with the same axis as the first; now (1) is

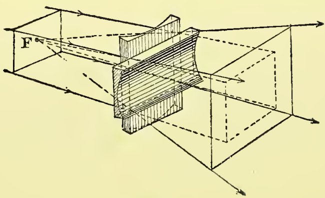

Fig. 36.

equivalent to a sphere with the same power as the second. Hence—

Two crossed cylinders of unequal power may be replaced by a sphere of power equal to that of the second combined with a

cylinder of power equal to the difference between those of the first and second, with the same axis as the first. And moreover either of the two cylinders may be regarded as the first or second.

The symbol ⊂⊃ is used to mean "combined with."

As an example let us take the case of the two cylinders + 7 **D** cyl. axis 70° ⊂⊃ + 4 **D** cyl. axis 160.° Now the + 7 **D** cyl. axis 70° may be considered as replaced by + 4 **D** cyl. axis 70° ⊂⊃ + 3 **D** cyl. axis 70°; we then have for the combination

+4 **D** cyl. axis 70° ⊂⊃ + 4 **D** cyl. axis 160° ⊂⊃ +3 **D** cyl. axis 70°

which is equivalent to

+ 4 **D** sph. ⊂⊃ + 3 **D** cyl. axis 70.°

Again regarding + 4 **D** cyl. axis 160° as the first lens, we get the combination equivalent to

+7 **D** cyl. axis 160° ⊂⊃ −3 **D** cyl. axis 160° ⊂⊃ +7 **D** cyl. axis 70°

that is

+ 7 **D** sph. ⊂⊃ − 3 **D** cyl. axis 160.°

The same rule is applicable to all cylindrical lenses, whether positive or negative.

Angle-reading for Cylindrical Lenses.—It is necessary to agree upon some plan of reading and registering the indication of the axis of cylindrical lenses. All authorities agree in starting from horizontal as the zero position. Some ophthalmic surgeons read always from zeros on the left, others always from zeros on the right; others read angles to the right on one eye of the patient, and to the left on the other eye, counting their degrees from the nose outwards. It is best to read always counter-clockwise, *i.e.* beginning at a zero on the observer's right, to read upwards. This is recommended by the Optical Standards' Committee.

43 TRANSPOSITION OF SPHERO-CYLINDER COMBINATIONS.

In constructing spectacle lenses to an oculist's prescription, more especially sphero-cylindrical lenses, the optician frequently finds that he is either unable to do so without a great deal of trouble, because the means of grinding at his disposal are limited, or that the lens if made exactly as prescribed would not

fit neatly into the frames, or for other practical reasons would be unsuitable. The question then arises as to whether or not other curvatures than those prescribed could be substituted, to produce the same optical effect. In a great many cases such transpositions are possible, and to explain these we will deal briefly with the underlying principles.

The subject has already been touched upon from one point of view when treating with crossed cylinders.

I. *Transposition of Simple Cylinders.*—It is clear from the consideration of simple spherical lenses, that two cylindrical lenses of equal and opposite powers, when placed together upon the same optic axes with their axes of figure parallel have, as a whole, no effect as a lens. Suppose then, in front of any given cylinder, we place such a pair of individual power numerically equal to that of the given cylinder, and with their axes perpendicular to its axis. Then the combined effect of the three is the same as that of the given cylinder alone; but we may regard them as two crossed cylinders of equal power, and a third of equal but opposite power with its axis perpendicular to that of the given cylinder. Now two crossed cylinders of equal and like power may be replaced by a sphere of equal and like power. We are then left with a sphere of power equal to that of the given cylinder, and a cylinder of equal but opposite power on an axis at right angles. Thus we arrive at the first guiding principle:—

Any given cylinder may be replaced by a sphere of the same power, and a cylinder of equal but opposite power on an axis at right angles.

Now what has been said regarding powers is equally true of curvatures [for the power of any refracting surface is its curvature multiplied by the constant $(\mu - 1)$]. Hence the rule may be paraphrased as—

Any given cylinder may be replaced by a sphere of equal numerical curvature, and a cylinder of equal but opposite curvature on an axis at right angles.

Example I.—How would you replace a + 6 **D** cyl. ax 5° **by** a sphere and a negative cylinder?

+ 6 **D** cyl. ax. 5° = + 6 **D** cyl. ax. 5° ○ + 6 **D** cyl. ax. 95° ○
− 6 **D** cyl. ax. 95° = + 6 **D** sph. ○ − 6 **D** cyl. ax. 95°.

Example II.—What are the spherical and cylindrical equivalents of a -3 **D** cyl. ax. 34°? *Answer.* -3 **D** sph. $+3$ **D** cyl. ax. 124°.

Example III.—It is required to replace a $+9$ **D** cyl. ax. 30° lens by means of another, one surface of which is to be spherical and the other cylindric. What are their respective curvatures? $\mu = 1 \cdot 5$.

The curvature of the given cylinder $= +\frac{9}{\cdot 5} = +18$

Hence the curvature of the spherical surface must be equal to $+18$, and its radius of curvature $\frac{100}{18} = 5\frac{5}{9}$ cms.

The curvature of the cylindrical surface $= -5\frac{5}{9}$ cms.

The above rules supply another way of finding the equivalents of crossed cylinders; thus, to find the equivalents of

$$+9\ \mathbf{D},\ \text{cyl. ax. } 7^\circ \bigcirc -4\ \mathbf{D},\ \text{cyl., ax. } 97^\circ.$$

Now we have

$$+9\ \mathbf{D},\ \text{cyl. ax. } 7^\circ = +9\ \mathbf{D}\ \text{sph.} -\bigcirc\ 9\ \mathbf{D},\ \text{cyl. ax. } 97^\circ.$$

Therefore

$$+9\ \mathbf{D},\ \text{cyl. ax. } 7^\circ \bigcirc -4\ \mathbf{D},\ \text{cyl. ax. } 97^\circ = 9\ \mathbf{D},\ \text{sph.} \bigcirc$$
$$-9\ \mathbf{D},\ \text{cyl. ax. } 97^\circ \bigcirc -4\ \mathbf{D},\ \text{cyl. ax. } 97^\circ = 9\ \mathbf{D},\ \text{sph.} \bigcirc -13\ \mathbf{D},\ \text{cyl. ax. } 97^\circ.$$

For purposes of reference the rules for transposition are given below (together).

44

TRANSPOSITION RULES.

1. *Simple cylindrical Lenses.*—Any given cylinder may be replaced by a sphere of the same power, combined with a cylinder of equal but opposite power on an axis at right angles; or,

Any given cylinder may be replaced by a sphero-cylinder, whose spherical surface has the same curvature, and whose cylindrical surface has an equal but opposite curvature about an axis at right angles.

2. *Crossed Cylinders.*—Two crossed cylinders of equal power may be replaced by a sphere of the same numerical power, and *vice versâ.*

Two crossed cylinders of unequal power may be replaced by a sphere of power numerically equal to that of the second com-

bined with a cylinder, of power equal to the algebraic difference between those of the first and second, with the same axis as the first; or,

Two crossed cylinders of unequal power may be replaced by a combination of a sphere and a cylinder in either of two ways:—

(*i*) A sphere of the same numerical value as the second of the cylinders, combined with a cylinder equal to the algebraic difference between the powers of the first and second cylinders, with the same axis as the first; or,

(*ii*) A sphere of the same numerical value as the first of the cylinders, combined with a cylinder equal to the algebraic difference between the powers of the second and the first cylinders, with the same axis as the second.

3. *Sphero-cylindrical Lenses.*—Any given sphero-cylindrical lens may be replaced either by a suitable pair of crossed cylinders or by a different sphero-cylindrical lens. The same principles of transposition will apply; for each spherical curvature may be resolved into a pair of equal crossed cylinders at any angle convenient for calculation.

For example, take the sphero-cyl. lens:—

+ 7·5 **D** sph. ⊂⊃ + 2 **D** cyl. vert.

This is equivalent to

+ 7·5 **D** cyl. vert. ⊂⊃ + 7·5 **D** cyl. hor. ⊂⊃ 2 **D** cyl. vert.,

whence, collecting together the vertical components, we find as the equivalent crossed-cylinder lens:—

+ 9·5 **D** cyl. vert. ⊂⊃ + 7·5 cyl. hor.

And this combination, by the rules just discussed, can again be transposed into

+ 9·5 **D** sph. ⊂⊃ – 2 **D** cyl. hor.

It will be observed that there are always possible *three combinations* to give any desired effect; and a prescription given in one form can always be transposed into one of the other two. The three forms are:—

1. Crossed cylinders.
2. Sphere with + cyl. (at some angle).
3. Sphere with – cyl. (at a right angle to the former).

To transpose from either of the sphero-cyl. forms to the other, all that is necessary is: give to the sph. an additional spherical power (+ or − as the case may be) equal to that of the cyl.; and change the cyl. to an equal cyl. of opposite type in a meridian at right angles to its former meridian.

Example 1.

+ 6 **D** sph. $\bigcirc$ + 2 **D** cyl. 5° transposes into
+ 8 **D** sph. $\bigcirc$ − 2 **D** cyl. 95°.

Example 2.

− 2·5 **D** sph. $\bigcirc$ + 4 **D** cyl. 25° transposes into
+ 1·5 **D** sph. $\bigcirc$ − 4 **D** cyl. 115°.

Example 3.

+ 5 **D** sph. $\bigcirc$ − 1·5 **D** cyl. 10° transposes into
+ 3·5 **D** sph. $\bigcirc$ + 1·5 **D** cyl. 100°.

Example 4.

+ 2·5 **D** sph. $\bigcirc$ − 7·5 **D** cyl. hor. transposes into
− 5 **D** sph. $\bigcirc$ + 7·5 **D** cyl. vert.

As there are always three possible alternatives, the question naturally arises which is the best to choose. Opticians generally avoid crossed cylinders, as sphero-cyls. are cheaper to grind; and, of the two cases of sphero-cyls., the preference seems to be for the form which, on calculation, results in having the smallest numerical curvatures (for instance, in Example 4, the first of the two), since deep curvatures always involve heavier lenses. Preference is also often given to that transposition which results most nearly in a periscopic form, with + sph. outside and − cyl. inside.

45 TOROIDAL LENSES OR TORIC LENSES.

Another way to construct lenses so that they have different refracting powers in different meridians, is to use a lens having the surface that is nearer to the eye a concave surface, and grind the other with a *toroidal* surface. A toroidal surface (Fig. 37) is one which has one radius of curvature in one meridian, and a different radius of curvature in the meridian

at right angles. "Toroid" is the scientific name for a surface like that of a bicycle tyre. Suppose a bicycle tyre 2 feet in diameter and 2 inches in apparent thickness laid down flat. On its outer surface the curvature is such that in the vertical meridian the radius is 1 inch; in the horizontal meridian the radius is 1 foot. If a tool of toroidal form, such as Fig. 37, were used to grind a lens, it would give a concave toroidal curvature, resembling a dish cover or spoon. The powers of such lenses in the two meridians of greatest and least curvature would be calculated simply by rules—

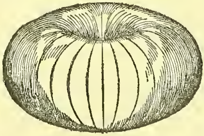

Fig. 37.
A Toroidal Solid.

$$D_1 = (\mu - 1)\frac{1}{r_1}, \quad \text{and} \quad D_2 = (\mu - 1)\frac{1}{r_2};$$

where $\mu - 1$ is the refractivity of the glass, and r_1 and r_2 the two radii of curvature. Save in the case of periscopic positive lenses for a hypermetropic astigmatic patient, all that any toroidal lens could possibly do could be done equally by using crossed cylinders, or by using the equivalent sphero-cyl. combination. There is, however, the great disadvantage of requiring special costly tools to grind each separate form of toroid. The name "toric" is less accurate than "toroidal" for lenses of this form.

46 OBLIQUELY-CROSSED CYLINDRICAL LENSES.

Another useless variation of form is that of using on the two faces of a lens two cylindrical surfaces crossed obliquely. The formulæ for obliquely crossed cylinders are somewhat complicated: but it can be shown that in no case can two obliquely-crossed cylinder lenses effect any optical purpose which cannot be equally well effected by an ordinary sphero-cylinder combination.

Sometimes, however, it becomes necessary to transpose into

a sphero-cylinder combination some case of obliquely-crossed cylinders: and for such the following formulæ are correct.

Let A and B be the powers (dioptries) of the given oblique cylinders, and θ the angle at which they are crossed. It is required to find the power X of the resultant cylinder, ϕ the angle that its axis makes with the axis of A, and Y the power of the resultant sphere. Then

$$\frac{X}{\sin 2\theta} = \frac{A}{\sin 2(\theta - \phi)} = \frac{B}{\sin 2\phi} \quad . \quad . \quad (1)$$

$$X^2 = A^2 + B^2 + 2\,A\,B \cos 2\theta \quad . \quad . \quad . \quad (2)$$

$$\sin 2\phi = \frac{B}{X} \sin 2\theta . \quad . \quad . \quad . \quad . \quad (3)$$

$$Y = \frac{A + B - X}{2} . \quad . \quad . \quad . \quad . \quad (4)$$

Example.—Find the sphero-cylinder equivalent of the following obliquely-crossed cylinders:—

+ 7 **D** cyl. ax. 20° ⬭ + 5 **D** cyl. ax. 35°.

The angle θ between them is evidently 15.° Hence

$$2\theta = 30° \text{ and } \cos 2\theta = 0{\cdot}866 \text{ and } \sin 2\theta = 0{\cdot}5.$$

Then, by formula (2)

$$X^2 = 49 + 25 + 70 \times 0{\cdot}866 = 134{\cdot}6,$$

$$X = +11{\cdot}66 \text{ dioptries cylindrical.}$$

To find ϕ by formula (3) we have

$$\sin 2\phi = \frac{5}{11{\cdot}6} \times 0{\cdot}5 = 0{\cdot}2155,$$

whence, from Table of Sines, Art. **3**,

$$2\phi = 12° \, 26',$$

$$\phi = 6° \, 13';$$

or since the first cylinder is already set at angle 20,° the angle of the resultant cylinder must be set at 26° 13.′

Lastly, by formula (3)

$$Y = \frac{+7 + 5 - 11{\cdot}6}{2} = +0{\cdot}2 \text{ dioptries spherical,}$$

Hence the final result for the equivalent lens is:—

+ 0·2 **D** sph. ⬭ + 11·6 **D** cyl. ax. 26° 13′.

To facilitate calculations of cylinder combinations, the following Table is added:—

θ	$\sin 2\theta$	$\cos 2\theta$	$\cot 2\theta$	$\sin^2\theta$	$\cos^2\theta$
0	0·0000	1·0000	$\mp\infty$	0·00000	1·00000
5	0·1736	0·9848	5·6713	0·00757	0·99241
10	0·3420	0·9397	2·7475	0·03014	0·96983
15	0·5000	0·8660	1·7321	0·06698	0·93296
20	0·6428	0·7660	1·1918	0·11696	0·88304
25	0·7660	0·6428	0·8391	0·17859	0·82138
30	0·8660	0·5000	0·5774	0·25000	0·74996
35	0·9397	0·3420	0·3640	0·32902	0·67109
40	0·9848	0·1736	0·1763	0·41319	0·58676
45	1·0000	0·0000	0·0000	0·50000	0·50000
50	0·9848	− 0·1736	− 0·1763	0·58676	0·41319
55	0·9397	− 0·3420	− 0·3640	0·67109	0·32902
60	0·8660	− 0·5000	− 0·5774	0·74996	0·25000
65	0·7660	− 0·6428	− 0·8391	0·82138	0·17859
70	0·6428	− 0·7660	− 1·1918	0·88304	0·11696
75	0·5000	− 0·8660	− 1·7321	0·93296	0·06698
80	0·3420	− 0·9397	− 2·7475	0·96983	0·03104
85	0·1736	− 0·9848	− 5·6713	0·99241	0·00757
90	0·0000	− 1·0000	$\mp\infty$	1·00000	0·00000
95	− 0·1736	− 0·9848	5·6713	0·99241	0·00757
100	− 0·3420	− 0·9397	2·7475	0·96983	0·03104
105	− 0·5000	− 0·8660	1·7321	0·93296	0·06698
110	− 0·6428	− 0·7660	1·1918	0·88304	0·11696
115	− 0·7660	− 0·6428	0·8391	0·82138	0·17859
120	− 0·8660	− 0·5000	0·5774	0·74996	0·25000
125	− 0·9397	− 0·3420	0·3640	0·67109	0·32902
130	− 0·9848	− 0·1736	0·1763	0·58676	0·41319
135	− 1·0000	0·0000	0·0000	0·50000	0·50000
140	− 0·9848	0·1736	− 0·1763	0·41319	0·58676
145	− 0·9397	0·3420	− 0·3640	0·32902	0·67109
150	− 0·8660	0·5000	− 0·5774	0·25000	0·74996
155	− 0·7660	0·6428	− 0·8391	0·17859	0·82138
160	− 0·6428	0·7660	− 1·1918	0·11696	0·88304
165	− 0·5000	0·8660	− 1·7321	0·06698	0·93296
170	− 0·3420	0·9397	− 2·7475	0·03014	0·96983
175	− 0·1736	0·9848	− 5·6713	0·00757	0·99241
180	0·0000	1·0000	$\mp\infty$	0·00000	1·00000

The amount of cylindrical effect which a cylinder set with its axis in any given meridian produces in any other meridian can be obtained by multiplying the power of the cylinder by the square of the cosine of the angle between the two directions. For example, if a + 7 **D** cyl. is set at 35° with the horizontal, the horizontal component of its cylindrical effect will be

$$+\ 7 \times \cos^2 35^\circ = +\ 7 \times 0{\cdot}671 = +\ 4{\cdot}7\ \mathbf{D}\ \text{cyl. hor.}$$

Similarly its vertical component will be expressed as

$$+\ 7 \times \cos^2 55^\circ = +\ 7 \times 0{\cdot}329 = +\ 2{\cdot}3\ \mathbf{D}\ \text{cyl. vert.}$$

If angles are reckoned from the horizontal, then horizontal components are proportional to $\cos^2$, and vertical components to $\sin^2$ of the angle. And the sum of the vertical and horizontal components *always* equals the original value of the cylinder.

47 PRISM FORMULÆ.

The following formulæ connect together the angle a between the faces of a prism, the angle δ of the deviation it produces, and μ the refractive index of the material.

Case I. *When the incidence is such that the light either enters or leaves normally to one surface,*

$$\sin a = \frac{\sin(a+\delta)}{\mu},$$

whence

$$\mu = \frac{\sin(a+\delta)}{\sin a}.$$

Case II. *When the incidence is such that the angles of incidence and emergence are equal,*

$$\mu = \frac{\sin \frac{1}{2}(a+\delta)}{\sin \frac{1}{2} a};$$

in this latter case the deviation is a minimum.

Example.—A flint-glass prism having angle 59° 56′ 22″, gave as the angle of minimum deviation, when using blue-green light (Fraunhofer's line F), 47° 35′ 59″. Calculating by the last formula, we find $\mu = 1{\cdot}61177$ for that kind of light.

In the case of *thin* prisms with small angles, so that arcs may be substituted for sines, both these formulæ become

$$\mu = \frac{a + \delta}{a}.$$

48 PRISMS FOR SPECTACLE WORK.

The prisms with which ophthalmic opticians are concerned are always *thin*, that is to say the angle between their refracting surfaces is small. This circumstance much simplifies calculations, enabling us to take angles simply instead of sines of angles into the calculations.

The one formula, in fact, may then be written—

$$\delta = (\mu - 1)\, a,$$

where a is the angle between the two faces of the prism, $\mu - 1$ the refractivity (in air) of the glass, and δ the angle of deviation produced by the prism.

Since for crown glass μ is about $1 \cdot 5$, $\mu - 1$ is approximately one-half. Hence, approximately, the deviation produced by a prism is half its own angle. A prism of $12°$ produces a deviation of about $6°$, for example.

The "Prism-Dioptrie."—The above simple relation has given rise to a notation for prisms similar to the dioptric system for lenses. Unit deviation in this system is called the "prism-dioptrie." *A prism is said to produce a deviation of one prism-dioptrie if it is such as to cause a ray to be deviated* 1 *centimetre from its original path, at a distance of* 1 *metre.* In fact 1 prism-dioptrie means a deviation of one per cent. This is most clearly understood by reference to Fig. 38.

Let P A be the path of a ray of light in air. If at Q a thin prism is interposed, this ray will be deviated slightly toward the base of the prism, and will travel say along the path Q B. Measure out from Q along the original path a length of 1 metre—represented here by the line Q A. Then measure the short lengths A B to see how much the ray has been deviated in

travelling the distance of 1 metre. If A B is one centimetre, then the prism had a power of one prism-dioptrie. If A B is two centimetres, then the prism had a power of two prism-dioptries, and so forth.

It is clear then that here is a way of describing the amount of angle of deviation, not expressing it in degrees of arc, but in terms of the length along a tangent scale placed at a distance

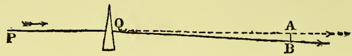

FIG. 38.

of 1 metre. Now the number of degrees of arc that correspond to 1 prism-dioptrie will obviously be such an angle that its natural tangent is equal to 1 per cent., or to 0·01. This is 0° 34′ 22″.

Prisms are used in spectacles to correct the tendency for the eye to look in a wrong direction. In extreme cases this tendency is called squinting or *strabismus:* but where the tendency is slight—some weakness of the side-muscles of the eyeballs being the cause — the defect is called *diplopia* (= double vision). To alleviate the tendency of the eyeball to look in a slightly oblique direction, a prism—of suitable angle—may be introduced. Or instead a decentred lens may be used.

49 Prismatic Effect of a Decentred Lens.

Let us consider the effect of a positive 1-dioptrie lens; it is diagrammatically represented in Fig. 39. Being a 1-dioptrie lens its principal focus will be 1 metre away on its axis, say

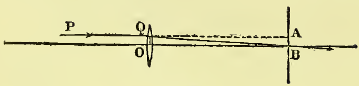

FIG. 39.

at B. First take the case of a ray, P Q, entering parallel to the axis and at a distance Q O, equal to one centimetre, away from the axis. Then P Q is deviated to B, the principal focus.

Now if a plane is drawn through B at right angles to the axis O B, and P Q produced to cut this in A, we see that A B = O Q = one centimetre; that is the deviation produced by the lens of 1 dioptrie on a ray entering one centimetre from the axis is the same as that due to a prism of one prism dioptrie.

Hence the angle of a prism having a power of one prism-dioptrie is the angle between the tangent planes to the two faces of a 1-dioptrie lens at points distant one centimetre from the axis, and on the same side of it.

The relation between prisms and lenses of numerically equal prismatic power for the ray under consideration is shown in Fig. 40.

Very frequently, and in fact most frequently, patients require not merely simple prisms but combinations of prisms and lenses,

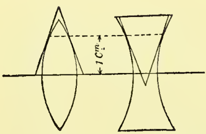

Fig. 40.

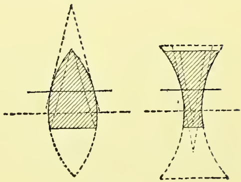

Fig. 41.

because, besides suffering from diplopia, they may also be suffering from short sight, long sight, or astigmatism. To meet such needs, prismatic lenses, such as are shown in an exaggerated form in the accompanying figure, may be used.

Now these prismatic lenses are portions cut from spherical lenses eccentrically; and it is obviously much easier to cut them from ordinary lenses than to go to the trouble of grinding a prism to correct the angle, and then superposing the necessary curvatures. It therefore becomes a necessity to have some method of determining the right place from which to cut a required prismatic lens, from a given spherical lens, in order to obtain the right prismatic effect.

Suppose we take a spectacle lens of one dioptrie (sph.)

power, such as shown diagrammatically in Fig. 42, with its principal axis A O B, and cut out a portion bounded by dotted lines, *p q r s*, so that its axis of figure, A′ O′ B′, shall be one centimetre from the original principal axis. Then from what has been said about the prism-dioptrie, it is clear that this portion would have a prismatic effect of one prism-dioptrie, the centre of the piece used being one centimetre from the axial centre. And clearly, whatever the power of a lens may be, if its axis of figure is decentred one centimetre from its optic centre, its prismatic power in prism-dioptries will be numerically equal to its power in ordinary dioptries.

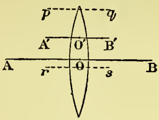

Fig. 42.

What will be the effect if it is decentred by some other amount? Without going into geometrical proofs, it may be stated that to a first approximation, it is accurate to say that the angle between the two tangent planes, and therefore the prismatic effect of decentring is proportional to the distance through which the lens is decentred laterally (i.e. the distances of the points of contact of the tangent planes from the principal-axis).

Let d = the amount of decentring in centimetres.
D = the power of the lens in dioptries.
Δ = the prismatic effect in prism-dioptries.

Then if $d = 1$ cm. and $D = 1$ dioptrie, we have as previously shown $\Delta = 1$ prism dioptrie.

Again,

If $d = 1$ cm. and $D = D$ dioptries
$\Delta = D$ prism-dioptries.
If $d = 2$ cm.
$\Delta = 2\,D$ prism-dioptries.
If $d = d$ cms.
$\Delta = d\,D$ prism-dioptries.

Or, if we wish to find the necessary amount of decentring, we may write the formula

$$d = \frac{\Delta}{D}$$

This formula is approximate only: it may be taken as correct for all lenses under 12 D.

An example will illustrate the use of the formula :—

The visual axes of a certain patient are found to have 4 prism-dioptries divergence, while he requires + 5 **D** lenses. How much must his 5 **D** lenses be decentred to correct the defect?

Clearly each must be decentred enough to produce 2 prism-dioptries. Then

$$d = \frac{\Delta}{\mathbf{D}} = \tfrac{2}{5} = 0{\cdot}4 \text{ cm.} = 4 \text{ mm.}$$

That is, the optical centre of each lens when cut must be 4 mm. from the centre of outline.

The same laws apply to diverging lenses, with the difference that while a + lens decentred gives an apparent deviation *against* the decentring, a − lens gives a deviation *with* the decentring.

50 To Convert Prism-dioptries to Degrees of Arc.

It is important to find a method of at once converting degrees of deviation into prism-dioptrie units, and *vice versâ.* In the first case, the tangent of 1° = 0·01745, which means that at a distance of one metre it subtends a length of 0·01745 metres or 1·745 centimetres on a scale placed square across, as shown in Fig. 43.

Thus, if Q A represents 1 metre, and the angle A Q B one degree, then $\tan AQB = \tan 1^\circ = \frac{AB}{AQ} = 0{\cdot}01745$.

Therefore $AB = 0{\cdot}01745 \times AQ$.

That is, $AB = 0{\cdot}01745$ metres $= 1{\cdot}745$ cms.

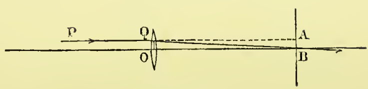

Fig. 43.

By the definition of a prism dioptrie, if A Q B were equal

to 1 prism-dioptrie, A B would be 1 centimetre. Hence, since small angles are proportional to their tangents, we have—

$$\frac{1 \text{ prism-dioptrie}}{1 \text{ degree}} = \frac{1}{1{\cdot}745},$$

or

$$1 \text{ prism-dioptrie} = \frac{1}{1{\cdot}745} \text{ degrees} = 0^\circ\ 34'\ 22''.$$

If the glass has index 1·54, and if the refractory angle be 1·85°, then $\delta = 1^\circ$; and this prism has $1{\cdot}745^\Delta$. So we may tabulate thus.

$$1^\circ = 0{\cdot}94^\Delta = 0{\cdot}54^\delta$$
$$1^\Delta = 1{\cdot}06^\circ = 0{\cdot}57^\delta$$
$$1^\delta = 1{\cdot}745^\Delta = 1{\cdot}85^\circ$$

51 DECENTRING EQUIVALENTS.

(From 'Opticians' Handbook.')

Lens.	To be equivalent to prisms of following edge-angles, decentre by number of millimetres given below [$\mu = 1{\cdot}54$].							
Dioptries.	1°	2°	3°	4°	5°	6°	8°	10°
1	9·4	18·8	28·3	37·7	47·2	56·5	75·8	95·2
2	4·7	9·4	14·1	18·8	23·6	28·2	37·9	47·6
3	3·1	6·3	9·4	12·6	15·7	18·8	25·3	31·7
4	2·3	4·7	7·1	9·4	11·8	14·1	18·9	23·8
5	1·9	3·8	5·7	7·5	9·4	11·3	15·2	19·0
6	1·6	3·1	4·7	6·3	7·9	9·4	12·6	15·9
7	1·3	2·7	4·0	5·4	6·7	8·1	10·8	13·5
8	1·2	2·3	3·5	4·7	5·9	7·1	9·5	11·9
9	1·0	2·1	3·1	4·2	5·2	6·3	8·4	10·5
10	·9	1·9	2·8	3·8	4·7	5·6	7·6	9·5
12	·8	1·6	2·4	3·1	3·9	4·7	6·3	7·9
14	·7	1·3	2·0	2·7	3·4	4·0	5·4	6·8
16	·6	1·2	1·8	2·4	3·0	3·5	4·7	6·0
18	·5	1·0	1·6	2·1	2·6	3·1	4·2	5·3
20	·5	·9	1·4	1·9	2·4	2·8	3·8	4·8

In this table the numbers given are the number of millimetres by which the lens (of given power) must be decentred in order to act as a prism of the angle mentioned at the top. For example:—In order that a lens of + **5 D** shall act as a prism of 2° edge angle it must be decentred 3·8 millimetres. For since 1^Δ deviates 10 mm. at 1 metre, 1 **D** must be decentred 10 mm. to produce the same effect as 1^Δ, and therefore must be decentred 9·4 mm. to produce same effect as glass prism having 1° edge angle.

52 Table giving Relation between Prism-dioptries and Degrees Deviation.

Prism-dioptries.	Deviation Angle Degrees.	Prism-dioptries.	Deviation Angle Degrees.
0	0	18	10° 12′
1	0° 34′ 22″	19	10° 46′
2	1° 8′ 40″	20	11° 20′
3	1° 43′	22	12° 25′
4	2° 17′	24	13° 30′
5	2° 52′	26	14° 35′
6	3° 26′	28	15° 39′
7	4° 0′	30	16° 42′
8	4° 35′	32	17° 45′
9	5° 8′ 30″	34	18° 47′
10	5° 43′	36	19° 48′
11	6° 17′	38	20° 48′
12	6° 51′	40	21° 48′
13	7° 24′	42	22° 47′
14	7° 58′	44	23° 45′
15	8° 32′	46	24° 42′
16	9° 5′	48	25° 38′
17	9° 39′	50	26° 34′

53 To Convert Prism-dioptries to Prism-angles.

It is also important that the relation between the prism-angle itself and its power in prism-dioptries should be known.

Call the number of degrees between the two faces of the prism A.

Then, since we are dealing with prisms of small angle only, we have

$$\delta = A\,(\mu - 1);$$

where δ is the number of degrees of deviation produced. But the number of degrees in a given angle is approximately equal to the number of prism-dioptries divided by $1 \cdot 745$.

That is,

$$\delta = \frac{\Delta}{1 \cdot 745}.$$

Hence

$$A = \frac{\Delta}{1 \cdot 745\,(\mu - 1)}.$$

And the formula for decentring may be written:—

$$d = \frac{A}{D} \times 1{\cdot}745 \times (\mu - 1) = \frac{\delta}{D} \times 1{\cdot}745 = \frac{\Delta}{D}.$$

If μ is taken as about 1·52, we get

$$A^\circ = \frac{\Delta}{0{\cdot}91}.$$

Or 1° of prism-angle produces a deviation of 0·91 prism-dioptrie. To give a deviation of 1 prism-dioptrie will therefore need a prism whose angle is 1·1 degree.

Hence the rule:—To convert prism-dioptries to degrees of prism-angle, divide by 0·91 or multiply by 1·1.

The three units therefore stand in the following relation:—

$$1 \text{ prism-dioptrie} = \frac{1 \text{ degree of deviation}}{1{\cdot}745} = 0^\circ\ 34'\ 22''.$$

$$= \frac{1 \text{ degree prism angle}}{0{\cdot}91}$$

The formulæ for decentring may now be written—

$$d = \frac{\cdot 91\ A}{D} = \frac{1{\cdot}745\ \delta}{D} = \frac{\Delta}{D}.$$

54 Obliquely-crossed Prisms.

If two (thin) prisms are crossed at an angle θ, they act as a prism of different power at an intermediate angle. Call the respective powers of the prisms A and B prism-dioptries; let the resultant prism be called R prism-dioptries, and let the angle it makes with prism A be called a. Then the values of R and a can be calculated by the following formulæ:—

$$R^2 = A^2 + B^2 + 2\,A\,B \cos\theta \quad . \quad . \quad . \quad \text{(i.)}$$

$$\sin a = \frac{B}{R} \sin\theta \quad . \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

Example.—Find the resultant of two prisms, one of 6 Δ at 60°, the other 4 Δ at 95°. Here $A = 6$, $B = 4$, $\theta = 35°$, whence $\cos\theta = 0{\cdot}819$, and $\sin a = 0{\cdot}573$. $R^2 = 36 + 16 + (2 \times 6 \times 4 \times 0{\cdot}819) = 91{\cdot}3$; whence $R = 9{\cdot}5$. Then $\sin a = 4 \times 0{\cdot}573 \div 9{\cdot}5 = 0{\cdot}24$; whence $a = 14°$. So the answer is that the resultant prism will be one of 9·5 Δ set 14° beyond the position of the prism A, that is to say, set at 74°.

55 Convergence of Vision. Metre-Angle.

Side by side with the prism-dioptrie system another method of measurement has been developed from the notion of angular convergence of the two eyes, and its unit is called the "*metre-angle.*" Before proceeding to define the metre-angle let us consider Fig. 44. Let $E\,E_1$ represent a pair of eyes, and also their centres. Join $E\,E_1$, bisect $E\,E_1$ in O, and consider a plane

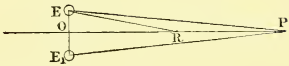

Fig. 44.

drawn through O perpendicular to $E\,E_1$, then O P is the line of intersection of this plane with the plane of the paper. This plane is called the median plane, and $E\,E^1$ the base line, while the two directions in which the eyes look are called the visual lines.

We can now define a metre-angle. It is the angle subtended by half the base line at a point on the median plane at a distance of one metre from either eye.

Thus in the figure if E P is equal to one metre the angle O P E is one metre-angle. The length of the base-line will clearly vary for different patients, and therefore the absolute value of the metre-angle will vary for each individual.

56 Table of Metre-Angles.

Base Line in Millimetres.	Sine of Metre-angle.	Value in Degrees and Minutes.
50	0·025	1° 26′
52	0·0255	1° 29′
54	0·027	1° 32′
56	0·028	1° 36′
58	0·029	1° 40′
60	0·030	1° 43′
62	0·031	1° 46′
64	0·032	1° 50′
66	0·033	1° 53′
68	0·034	1° 57′
70	0·035	2° 0′

57 TABLE OF ABERRATIONS IN LENSES.

—	Aberration.	Manifestation.	Remedy.
Class I.	DUE TO THE MATERIAL.		
A	Chromatic aberration— (*a*) Of focal plane .. .. .. (*b*) Of magnification	Colour tinging edges of images and of visible field	Use achromatic lens made of two kinds of glass. Partial remedy: stop down with diaphragm.
B	Secondary spectrum	Faint colour tinge in images of minute bright objects ..	Use apochromatic lenses made of three kinds of glass; or special achromatics of glass chosen to annul secondary spectrum.
C	Streakiness	Streaks in glass. Images of bright points confused	Reject. Glass is bad.
D	Double refraction..	Images doubled	Reject. Cause: crystalline structure or bad annealing.
Class II.	DUE TO THE FORM.		
E	Cylindricity	Vertical and horizontal lines not in focus at same time ..	Reject. Lens has got astigmatism, being badly ground But sometimes due to oblique mounting.
F	Spherical Aberrations— (*a*) Central .. .. (*b*) Coma .. .. (*c*) Radial astigmatism	 Images of bright points blurred, even at centre of field .. Image of points not at centre of field blurred into pear shape Image of points not at centre distorted into two focal lines at different distances from lens.	Partial remedy: stop down. Curves badly calculated. In telescopes, correct by retouching to correct for error arising from surface being truly spherical. Other remedy: use separated component lenses, or recalculate curves, so that work of refraction shall be shared between various surfaces.
G	Chromatic differences of the Spherical Aberration.	Faint colour tinge of blurred image	Remedies and corrections as preceding.
H	Curvature of Focal Plane	Images at centre and at margin are in focus at different distances.	Different combinations of lenses needed.
I	Distortion of image in its plane..	Barrel distortion or pin-cushion distortion of image	Combination badly designed. Try shifting stop.
Class III.	DUE TO APERTURE.		
K	Diffractional aberration	Spurious disks of stars; sometimes faint symptoms of rings round star.	
Class IV.	DUE TO BAD MOUNTING.		
L	Obliquity of mounting	Vertical and horizontal lines not in focus at same time ..	Set straight in mounting.
M	Decentring	Image shifted out of direct line	Lens badly cut or mounted. Thickest part must be in centre.
Class V.	DUE TO STRAY REFLEXION.		
N	Flare spot	Flare intruding in middle of field	Combination not well adjusted. Alter distance between lenses.

58 NOTES ON ABERRATIONS OF LENSES.

Any lens which will bring light that emanates from a point on one side of the lens to focus *accurately at a single point* at the other side of a lens is described as accurately *stigmatic* (*stigma* means a *point*). Any failure of a lens to produce perfectly definite and undistorted images is called an *aberration*. See Table **57**.

CHROMATIC ABERRATION.—Every single kind of glass has a greater refracting effect on blue light than it has on red light. The production of the spectrum colours by glass prisms proves that each kind of colour of light is affected differently. For each different colour the refractive index is different. Hence any and every lens made of any single kind of glass will have different powers for lights of different colours, and will therefore have different focal lengths for light of different colours. The focal length is always shorter for blue than for red light.

Example of Chromatic Aberration.—Consider a biconvex lens, made of the kind of flint glass mentioned last in Table **18** (Chance's glasses). Let its radii of curvature r_1 and r_2 be each 200 millimetres, or 0·200 metre, i.e. the curvature of each face is 5 dioptries. The total surface curvature is 10 dioptries. If we multiply this by the refractivity we get the power. Now this glass has refractive index 1·6965 for red ("A"), 1·7129 for yellow ("D"), and 1·7466 for blue ("G"). Its corresponding refractivities will be 0·6965, 0·7129, and 0·7466 respectively. Hence the lens will have the following powers:—for red light 6·9 dioptries; for yellow 7·1 dioptries; and for blue light 7·4 dioptries. So its focal lengths will be for red light 144·9, for yellow 140·8, and for blue 135·2 millimetres. The focus for blue light is 9·7 millimetres nearer the lens than that for red light.

59 ACHROMATIC LENSES.

If the dispersion (i.e. the difference between the refractions for different colours) were always proportional to the refraction there could be no remedy for the chromatic aberration. But inspection of any of the Tables of Refractive Indices will show that this is never so. In these Tables the refractive index for mean light—i.e. yellow light of the sodium "D"-line—is given

under the heading D, or μ_D. Also the medium dispersion is given, that is to say the difference between μ_C, the refractive index for red light of the quality of the "C"-line, and μ_F, the refractive index for blue-green light of the "F"-line of the spectrum. This is written $\mu_F - \mu_C$ or for brevity $\Delta\mu$, meaning the difference between the μ's. Now if this dispersion were always proportional to the mean refraction $\mu_D - 1$, the fraction $\frac{\mu_D - 1}{\mu_F - \mu_C}$ or $\frac{\mu_D - 1}{\Delta\mu}$ would be the same for all glasses.

A glance at the last column of any of the Tables of Refractive Indices will show at once that so far from this being the case the numbers vary widely. This ratio is all-important in the practical calculation of lenses; it states *the amount of refraction for a given amount of dispersion*. and is often denoted by the symbol ν. We see that in the lightest kind of pure glass, Table [**16**], ν is worth 70, while in the heaviest (flint) kind it is worth 19·7. The values of ν in Chance's glasses range from 64·6 to 29·9. Now it is obvious, on a little thought, that if we want to so combine two lenses that they shall neutralise one another's dispersion, two conditions must be fulfilled:—(*a*) the lenses must be of opposite kinds, one + the other − ; (*b*) their refracting powers must be so chosen that the refracting lens of greater refracting power shall produce exactly as much dispersion as the lens of lesser refracting power. This last provision clearly implies that the respective powers of the two lenses chosen shall be proportional to their respective values for ν.

Understanding this, nothing is easier than to make an achromatic lens. For example, in Chance's list of glasses there is a "hard crown" for which ν is 60·5, and an "extra dense flint," for which ν is 29·9. The crown gives almost exactly twice as much refraction for an equal dispersion as compared with the flint. Hence, if we take a + lens made of this crown, of a power proportional to 60·5, and a − lens made of this flint of a power proportional to 29·9, they will have equal and opposite dispersions. Let us then take a plano-convex crown of + 6·05 **D** and a plano-concave flint of − 2·99 **D**, and cement them back to back like Fig. 45. They will make an achromatic lens of a power of + 3·04 **D**. Again, in Chance's list there is a "dense flint," for which $\nu = 36$. If we were to

use this, instead of the extra-dense flint, along with the hard crown, the refracting powers of the two components must be in the proportion of crown + 60·5 to flint − 36, to make a positive achromatic lens. For a negative achromatic lens the proportions would have to be:—crown − 60·5 to flint + 36. The difference between 60·5 and 36 is 24·5. Suppose we wished to make a positive achromatic lens having a focal length of 24 inches. This will be of 1·64 dioptries power. Then the crown must be such that its power is to 1·64 as + 60·5 is to 24·5; and the flint must be such that its power is to 1·64 as − 36 is to 24·5. Or

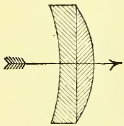

Fig. 45.

$$\text{Crown lens} = \frac{+60\cdot5}{24\cdot5} \times 1\cdot64 = +4\cdot08 \text{ dioptries}$$

$$\text{Flint lens} = \frac{-36}{24\cdot5} \times 1\cdot64 = -2\cdot44 \text{ dioptries}$$

$$\text{Resulting achromatic lens} = +1\cdot64 \text{ dioptries}$$

If the crown and flint components are both plano-lenses stuck back to back as in Fig. 45, the combination constitutes an achromatic form of cemented lens, which by itself is not very good, since it suffers from spherical aberration and distortion of field. Two such lenses, however, placed facing one another a little distance apart (Fig. 46), form an excellent

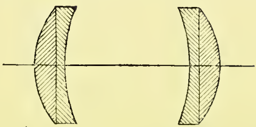

Fig. 46.

lens, with very little distortion of field. Two other forms, sketched in Figs. 47 and 48, have been preferred in general; Fig. 47 for telescope objectives, Fig. 48 for camera lenses, as having less spherical aberration than the form made up of two

plano They require the calculation of 3 radii of curvature In the case of the lens just calculated, it is easy to find the radii

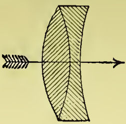

FIG. 47.

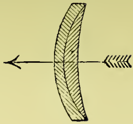

FIG. 48.

of curvature. The formula for the power of any lens (see Art. **28**) is:—

$$\text{Power} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

Now, as each of the component lenses has a flat face one of the two curvatures is = 0, and the formula becomes:—

$$\text{Power} = (\mu - 1)\frac{1}{r},$$

or

$$r = (\mu - 1) \div \text{power}.$$

The μ here is of course the mean refractive index for the particular kind of glass. Referring to the Table [**18**] of Chance's glasses we find:—

Hard crown, $\mu_D = 1 \cdot 5175$,
Dense flint, $\mu_D = 1 \cdot 6225$,

whence

r for crown lens $= 0 \cdot 5175 \div 4 \cdot 07 = 0 \cdot 127$ metre $= + 5$ inches.

r for flint lens $= 0 \cdot 6225 \div - 2 \cdot 43 = - 0 \cdot 256$ metre

$= - 10 \cdot 08$ inches.

New Achromats.—A new kind of achromatic combination was devised in 1892 by P. Rudolph. This consists in using for the crown-glass positive lens one of the new Jena glasses having a *higher* refractive index but a *lower* dispersion than the flint glass of the negative lens. For example, using the glass called "O 30" in Table **16** for the positive lens along with the flint glass called "O 726" in the same Table. These *New Achromats* give a flatter field than the old achromats.

Correction of Secondary Spectrum.—In the calculations of achromatic lenses we have taken as the dispersion of the glass its medium dispersion between lines "C" (red) and "F" (blue-green) of the spectrum. The combined lens is therefore corrected for these two colours, which are brought accurately to the same focus. But it does not necessarily follow that because these two are brought to one focus that the rays of all other colours will be. There are the yellow and green of intermediate wave-length; and there are of lesser wave-length the blue, violet, and ultra-violet waves. The eye is most sensitive to the former; the photographic plate to the latter. Will they all come to the same focus? The answer is that they do not accurately come to the same focus as the two rays for which correction has been made; there is a residual aberration called *secondary spectrum.* The failure of the correction is due to the circumstance that in every kind of glass the dispersion is not uniformly distributed in different parts of the spectrum. Look at the Table **16** of Jena glasses and you will find an ordinary silicate crown (Factory number "O 203"), and below it another silicate crown ("O 598"), which are very similar glasses. But on comparing them for their partial dispersions it will be seen that they do not disperse the colours equally. In the red-orange part of the spectrum the "ordinary" sort has a slightly higher dispersion 0·00563 than the other 0·00562; while in the yellow and green part the "ordinary" has a lower dispersion 0·00616 than the other 0·00619. The "ordinary" sort produces a spectrum in which the D-line in the yellow is shifted, relatively, further away from red towards the blue end of the spectrum. This "irrationality of dispersion" runs through the whole of the refracting materials available, and compels a study of the partial dispersions as well as of the dispersion as a whole. When such detailed information is available it is then possible to pick out from the list a pair of kinds of glass such that in whatever irrational way one of them distributes its dispersion the other shall be a glass that shows a very similar irrationality in its dispersion; so that as far as possible the two irrationalities may annul one another. Such a specially chosen achromatic combination will have very little secondary colour-error. The two

glasses O 60 and O 164 of Table **16** will, for example, if used as a pair, practically give no secondary spectrum.

Another important point in selecting the two kinds of glass is to see that the crown shall have a high refractivity relatively to its dispersion; and the flint the lowest possible. In other words, the crown should have the largest possible value of ν, and the flint the lowest. That is why these tables are arranged with the materials in the order of their values of ν. By following this last rule only a very thin flint will be needed to correct the dispersion of the crown; and the whole combination will be lighter as well as freer from secondary colour-error.

Apochromatic Lenses.—An achromatic lens made of two materials can only give perfect colour-correction for two parts of the spectrum. Herschel, Hastings, and others have proposed to construct lenses of three different materials, so as to correct for three regions of the spectrum. Lenses composed thus of three (or more) different materials, if also spherically corrected for two different colours, are for distinction called *apochromatic* lenses. In some of Zeiss's microscope objectives the third material is *white fluor spar*, which (see Table **23**) has a very low dispersivity, making its refractivity for a given relative amount of dispersion equal to 97, a value higher than that of the lightest crown glass.

60 SPHERICAL ABERRATION.

Suppose we were working with light of one colour only, so that no question could arise as to lights of different colours not focussing, we should yet find that however perfectly the lens was ground to perfect sphericity of figure, it would still fail to give perfect focussing; the rays would not all come to one point. those passing through the marginal zones of the lens crossing the axis nearer in than those that come through the central region of the lens. In Fig. 49 is represented, exaggerated, the case of parallel light passing through a plano-convex lens. The central rays have their focus at F, the marginal rays at J. Between the two is the place where the rays form the narrowest bundle. Here they form a small round spot—the "circle of least confusion"—the nearest approach to a point focus. To

sharpen the definition a diaphragm might be introduced to cut off all the marginal light; then the central rays would meet sharply at F. But this involves loss of light. How then, without covering up any part of the lens can *all* the light be made to converge accurately to F? The marginal parts refract too much. Suppose the curvature flattened at the outer parts by grinding

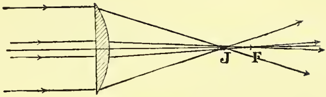

Fig. 49.

down the curve. A form may be found which will answer the requirement. But obviously this form will be no longer spherical. It will be more nearly ellipsoidal or hyperboloidal. This is one way of altering a lens so as to get rid of aberration inseparable from sphericity. In the case of large telescope objectives, they are first ground spherical, then *retouched* by hand on the marginal parts, polishing them away wherever necessary to make them perfect in their focussing effect.

The more that any ray in passing through the surface is inclined toward that surface, the greater is the aberration of that ray likely to be. The more nearly a ray passes normally through the surface, the less aberration will it suffer. Hence, it

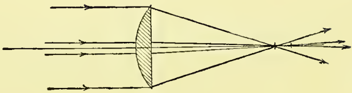

Fig. 50.

becomes a guiding principle that spherical aberration can be reduced to as little as possible by making such arrangements that the rays should *not* (as in Fig. 49) undergo violent refraction at one surface of the lens and none at the other, but such that at both surfaces about an equal amount of refraction should take place. By merely turning the plano-convex round so that its flat face is toward the focus, the aberration is considerably

lessened, the work of refraction being now shared between the two surfaces. Of all lenses of equal power that might be designed for carrying parallel light to or from a focus, the best form (if μ be taken at $1 \cdot 5$) is a biconvex, but not equiconvex, lens having the curvature of the face nearer the focus, one-sixth as great as that of the other face. Such a lens is called a "crossed" lens. If f be the focal length and y the semi-diameter of the aperture of the lens, the longitudinal aberration is—

$$\text{In a "crossed" lens} = -\frac{15}{14}\frac{y^2}{f},$$

$$\text{In an equiconvex lens} = -\frac{15}{3}\frac{y^2}{f}.$$

A lens which has been corrected for spherical aberration for two conjugate points on the axis, if all its zones have equal magnifying power, is called an *aplanatic* lens.

In the rule given above the amount of the longitudinal aberration is given as proportional to the square of the semi-diameter of aperture; but the *lateral* aberration increases with the *cube* of the semi-diameter.

A lens that has been made perfectly aplanatic for parallel light passing through it in one direction, will show spherical aberration for parallel light passing in the opposite direction, or also for light not parallel passing from one conjugate focus to another. In the case where the lens is required to produce an image of equal size to the object, and where therefore the conjugate points of object and image are equidistant from the lens, the shape of lens that will give least spherical aberration is the equiconvex.

Though it is necessary to correct telescope and other lenses for fine optical work for both spherical and chromatic aberrations, the latter is really much more important. In an equiconvex lens, for example, if of crown glass, the aberration of prismatic colour is sixteen times greater than the aberration of sphericity. If it is of flint glass, the aberration of colour is twenty-seven times as great as the aberration of sphericity. In a large telescope object-glass made by Frauenhofer, having focal length of 2 metres, and a semi-aperture of 66 millimetres, the longitudinal aberration due to sphericity was 4 millimetres (or might have been as little as $2 \cdot 2$ millimetres if the "crossed" form had been adopted). It was of course an achromatic pair

of crown and flint. But had it been of crown alone the chromatic aberration would have been about 60 millimetres, or if of flint alone about 100 millimetres!

61 OPTICAL INVARIANT.

Every spherically curved optical surface, such as the surface of a lens or the cemented surface between two lenses, possesses a definite optical property in respect of the refractive effect which it produces on a ray that meets it. It will be said that the refractive effect must vary for rays that strike at different points of the surface or at different angles. This is true. Nevertheless, it is possible to find an expression which holds good for *any* ray striking at *any* point of the surface. Let μ be the refractive index of the first medium, μ' that of the second medium, r the radius of curvature of the surface separating them; α the angle at which the ray is incident at any point of this surface, and α' the angle at which it passes on through the second medium, both being measured from the normal through the point of incidence. Then since $\sin\alpha/\sin\alpha' = \mu'/\mu$, we have $\mu\sin\alpha = \mu'\sin\alpha'$. This expression multiplied by the curvature $1/r$ of the surface, is called by Abbe the *Optical Invariant* of the surface, and denoted by the letter Q.

$$Q = \mu\frac{\sin\alpha}{r} = \mu'\frac{\sin\alpha'}{r}.$$

Apply this to the case of an oblique ray passing from a point on the axis to its conjugate point on the axis in the other medium. Let u and u' be the respective distances of these points from the vertex of the surface and p and p' the respective oblique distances from the point where the ray crosses the surface. Then the relation becomes:—

$$Q = \mu\frac{u-r}{pr} = \mu'\frac{u'-r}{p'r}.$$

If the angle which the radius, at the point where the ray crosses the surface, makes with the axis is small, then, ap-

proximately, $p = u$, and $p' = u'$; so that for axial rays we may write:—

$$Q_0 = \mu \frac{u - r}{ur} = \mu' \frac{u' - r}{u'r};$$

whence

$$\frac{\mu'}{u'} - \frac{\mu}{u} = \frac{\mu' - \mu}{r}.$$

If the first medium is air, for which $\mu = r$, we have:—

$$\frac{\mu'}{u'} - \frac{1}{v} = \frac{\mu' - 1}{r}.$$

Or, the more general formula may be written:—

$$\Delta \left(\frac{\mu}{u}\right) = \frac{1}{r} \Delta \left(\mu\right).$$

62 Spherical Aberration for Oblique Pencils.

The preceding Article **60** dealt with the effect of spherical aberration upon light passing direct along the principal axis of the lens. But pencils of light which pass obliquely through

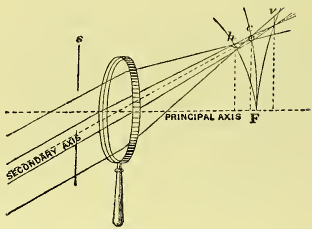

Fig. 51.

the lens suffer another kind of spherical aberration which in the case of camera lenses is much more important.

When a parallel beam is sent *obliquely* through a convex lens it is no longer brought to a point focus. Indeed, the effect of sending light obliquely through a lens is the same as if while the light was going straight the lens had been tilted. The general effect is the same as if there had been interposed a

cylindrical lens (compare Fig. 34). The result is that there now two focal lines instead of the one focal point. Suppose tn. oblique secondary axis of the beam to slope upwards, as in Fig. 51. The nearer focal line *h* will be horizontal (or tangential), and the further focal line *v* will be vertical (or radial). Between them there will be a "circle of least confusion" *c*, a sort of ill-defined focal patch. The less oblique the beam, the nearer together, and the smaller will be *v*, *c*, and *h*; while, for a beam moving exactly along the principal axis, all three are merged into one focal point at F. These three sorts of foci *v*, *c*, and *h* lie each on its own curved focal surface. A focussing screen placed at F will not receive sharp images of any luminous point that lies widely away from the axis. If the screen is pushed forward toward *h* it will show the image of a bright point as a short tangential line, while if put further away it will show it as a short radial line.* Somewhere in between will be the position that gives least blur; but the images will all be fuzzy. No lens can be of much service for a wide field of view—as in a camera—unless this kind of aberration is annulled.

This curvature of focal planes and want of definition all over the margins of the focussing screen can be partly corrected by putting a stop in front of the lens; for instance, as at *s*. And by moving this stop nearer to the lens or further away, the position of *c* can be altered. By putting *s* further away, *c* is caused to move slightly further away on the other side. By choosing the best position for the stop, the focal surface can be made practically a flat plane passing through the principal focus F.

But now another source of aberration makes itself seen, for the images on the screen appear distorted at the edges. This also depends on the position of the stop. A stop in front of a lens tends to make the image of a square object "barrel-shaped," the corners not being magnified enough; while a stop behind

* Some writers call this effect "astigmatism;" but the term should be "radial astigmatism," as it is a totally different thing from the astigmatism of the eye. If a lens had the defect of astigmatism, it would fail *all over the field* to show vertical and horizontal lines in focus at the same time.

the lens tends to make the image "cushion-shaped," the corners being unduly extended.

The remedy is *not* to put stops both in front and behind, but to construct the lens of two separated parts (each an achromatic pair) and put the stop in between them, as in all modern wide-angled camera lenses.

63 ENTRANCE-PUPIL AND EXIT-PUPIL.

It will be at once evident from the figure, that if the stop is placed symmetrically with respect to two similar lenses (on the same axis), any ray which passes through its centre will meet the lenses at the same angle and at corresponding points, and as

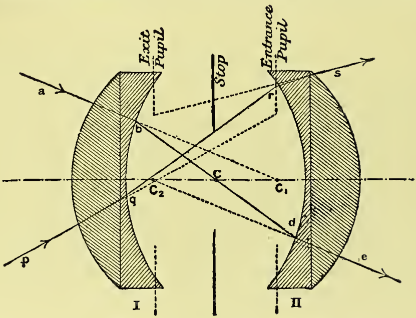

Fig. 52.

a consequence the portions of its path beyond the outer surfaces of the lenses are similarly situated with respect to the axis, and parallel to each other; hence if each lens is spherically corrected for the centre of the stop, each point of the object will send through it one ray, which is the "chief" ray or core of the oblique pencil as limited by the stop; and therefore the image must be geometrically similar to the object. Such a ray is drawn as *a*, *b*, *d*, *e* in Fig. 52. The use of this central stop has

been the cause of a new conception, introduced by Professor Abbe, of Jena. It is clear that each lens will form a virtual image of the stop, which may be seen by an eye looking into either lens. If the positions and sizes of these stop-images are ascertained, a simple geometrical construction will enable the position and size of the image of any given object to be predicted. Professor Abbe has given the name *Eintritts und Austritts-pupille* to these stop-images, terms which may be translated as *Entrance and Exit-pupils.* They are shown (dotted) with their centres at C_1 and C_2; C_1 being the image of the actual stop C due to component I, and C_2 the image due to II. A ray passing through the centre C of the actual stop is lettered $a\,b\,d\,e$; the outer portions $a\,b$ and $d\,e$ being parallel and directed towards the centres c_1 and c_2 of the entrance and exit pupils. Also $p\,q\,r\,s$ represents a ray grazing the edge of the actual stop; its outer portions $p\,q$ and $r\,s$ are not parallel but are directed to corresponding points on the edges of the entrance and exit pupils c_1 and c_2. When the two components I and II are dissimilar the entrance and exit pupils will be neither of the same size nor at equal distances from the actual stop, but the actual values of these quantities may be varied by varying the position of the stop relatively to the two components.

64 ABERRATION DUE TO APERTURE.

In telescopic optics there enter some further considerations as to the dependence of "definition" (i. e. sharpness of focus) upon the aperture and on the wave-length of light as well as upon the figure of the lens. According to Lord Rayleigh, the greatest permissible longitudinal aberration z is connected with the wave-length λ and with the angle of semi-aperture α by the relation :—

$$z = \frac{\lambda}{\alpha^2}.$$

But for a single lens, in the best case,

$$z = f \times \alpha^2,$$

whence

$$a^4 = \frac{\lambda}{f},$$

or the angle of semi-aperture ought not to exceed the value given by

$$a = \left(\frac{\lambda}{f}\right)^{\frac{1}{4}}.$$

Example.—An object-glass of 36 inches focal length. Assume for mean wave-length of light $\lambda = \frac{1}{50000}$ inch. Then a ought not to exceed $(\frac{1}{720000})^{\frac{1}{4}}$, or, approximately $\frac{1}{34}$ radian. As $f = 36$ inches, the largest object-glass that will be of service for accurate work will be about 1 inch in semi-aperture, or 2 inches in diameter. With any larger glass of same focal length the interference of the waves will make the definition worse.

The cause of this aberration is the interference of the waves that reach the same part of the image after traversing paths of different lengths through different zones of the lens. It is impossible here to enter further upon this complicated topic.

The diffraction-effects observed in microscopy are essentially also due to interference. Because of interference between the portions of the wave-fronts that emerge through parts of the last lens-surface that are wide apart from one another, the image which the lens forms of a luminous point is not itself a point, but is a small disc. Let the diameter of this spurious disc be called δ, and its distance from the lens be called v, the radius of aperture of the lens being called r. Then the angle of semi-aperture a will be such that $\tan a = r \div v$; and it also can be shown from interference-principles that $\tan a = \lambda \div 2\,\delta$. It follows that

$$\delta = \frac{\lambda}{2 \tan a} = \frac{v\lambda}{2r}.$$

This shows that the spurious disc will be smaller if the light to illuminate the object be of shorter wave-length, such as green or blue. Also that the objective if it is to resolve minute points, must have a wide angular aperture, otherwise the images of those points will be overlapping discs. See Table 81.

65 LINEAR MAGNIFYING POWER of Lenses of various Focal Lengths when used as MAGNIFYING GLASSES. The Distance of Distinct Vision being 10 inches.

Focal length of Lens.	Distance of Lens from the Eye in inches.						
	½	1	2	3	4	5	6
¼	39	37	33	29	25	21	17
½	20	19	17	15	13	11	9
1	10·5	10	9	8	7	6	5
2	5·75	5·5	5	4·5	4	3·5	3
3	4·167	4	3·67	3·33	3·00	2·67	2·33
4	3·375	3·25	3·00	2·75	2·5	2·25	2·0
5	2·9	2·80	2·60	2·40	2·20	2·00	1·80
6	2·58	2·50	2·33	2·167	2·00	1·833	1·67
8	2·19	2·125	2·00	1·875	1·75	1·625	1·50
10	1·95	1·90	1·80	1·70	1·60	1·50	1·40
12	1·79	1·75	1·66	1·583	1·50	1·42	1·33
15	1·63	1·6	1·53	1·47	1·40	1·33	1·27
20	1·475	1·45	1·40	1·35	1·30	1·25	1·20
24	1·396	1·375	1·33	1·29	1·25	1·21	1·17

Linear Magnification by a Lens used as a Magnifying Glass.

This table is calculated by the following formula $G = 1 + \frac{v}{f}$, where v is the distance from the lens to the virtual image, f the focal length of the lens, and G the linear magnification (i. e. the ratio of the actual size of the image to that of the object). If p be the distance of the near point of good distinct vision of the eye, and d the distance of the lens from the eye,

$$v = p - d.$$

65a Case II.—Distance of Distinct Vision being 12 inches.

Focal length of Lens.	Distance of Lens from the Eye.							
	½	1	2	3	4	5	6	7
¼	47	45	41	37	33	29	25	21
½	24	23	21	19	17	15	13	11
1	12·5	12	11	10	9	8	7	6
2	6·75	6·5	6	5·5	5	4·50	4·0	3·5
3	4·83	4·67	4·33	4·00	3·67	3·33	3	2·67
4	3·875	3·75	3·50	3·25	3·00	2·75	2·5	2·25
5	3·30	3·20	3·00	2·80	2·60	2·4	2·2	2·00
6	2·92	2·83	2·66	2·50	2·33	2·167	2·0	1·83
8	2·44	2·375	2·25	2·125	2·00	1·875	1·75	1·625
10	2·15	2·10	2·00	1·90	1·80	1·70	1·60	1·50
12	1·96	1·917	1·83	1·75	1·67	1·583	1·50	1·42
15	1·77	1·73	1·67	1·60	1·53	1·46	1·39	1·32
20	1·575	1·55	1·50	1·45	1·40	1·35	1·30	1·25
24	1·48	1·46	1·42	1·375	1·33	1·29	1·25	1·21

66 APPARENT MAGNIFICATION by Magnifying Glass.

Owing to the circumstances that the virtual image is situated further from the eye (at the distance p of the near point of distinct vision) than the object, the apparent magnification A is always less than the linear magnification G. In the preceding Table it is shown that G always decreases if the distance d of the lens from the eye is increased. But the apparent magnification is increased (as the simplest observation with a magnifying glass will show) as the lens is moved away from the eye. The greatest apparent magnification occurs when d is made equal to $\frac{1}{2} p$, and at that position the apparent magnification A has, as its maximum value, the value $= 1 + \frac{1}{4}\frac{p}{f}$. For all distances d that are either greater or less than $\frac{1}{2} p$, the apparent magnification is less than this. The values of A in the Table below are calculated in inches from the formula

$$A = 1 + \frac{d\,(p - d)}{f p}.$$

Apparent Magnification by Magnifying Glass.
Case I.—Distance of Distinct Vision (p) = 10 inches.

Focal Length (inches).	Distance of Lens from Eye.						
	½	1	2	3	4	5	6
¼	2·9	4·6	7·4	9·4	10·6	11·0	10·6
½	1·95	2·8	4·2	5·2	5·8	6·0	5·8
1	1·475	1·9	2·6	3·1	3·4	3·5	3·4
2	1·257	1·45	1·8	2·05	2·2	2·25	2·2
3	1·158	1·3	1·53	1·7	1·8	1·833	1·8
4	1·119	1·225	1·40	1·525	1·6	1·625	1·6
5	1·095	1·18	1·32	1·42	1·48	1·500	1·48
6	1·079	1·15	1·27	1·35	1·40	1·417	1·40
8	1·058	1·112	1·20	1·263	1·30	1·312	1·30
10	1·047	1·090	1·16	1·21	1·24	1·25	1·24
12	1·040	1·075	1·13	1·175	1·20	1·208	1·20
15	1·032	1·060	1·107	1·13	1·16	1·167	1·16
20	1·023	1·045	1·08	1·10	1·12	1·125	1·12
24	1·020	1·038	1·065	1·087	1·10	1·104	1·10

66a Apparent Magnification by Magnifying Glass.
Case II.—Distance of Distinct Vision 12 inches.

Focal Length (inches).	Distance of Lens from Eye.							
	½	1	2	3	4	5	6	7
¼	2·883	4·7	7·6	10	11·67	12·7	13·0	12·7
½	1·942	2·85	4·33	5·5	6·67	6·85	7·0	6·85
1	1·471	1·925	2·67	3·25	3·67	3·925	4·0	3·925
2	1·235	1·4625	1·83	2·125	2·33	2·462	2·5	2·462
3	1·157	1·308	1·56	1·72	1·89	1·975	2	1·975
4	1·118	1·231	1·42	1·56	1·67	1·731	1·75	1·731
5	1·096	1·183	1·33	1·45	1·53	1·583	1·6	1·583
6	1·078	1·154	1·27	1·37	1·44	1·487	1·5	1·487
8	1·059	1·115	1·202	1·26	1·33	1·365	1·375	1·365
10	1·041	1·092	1·166	1·225	1·27	1·292	1·30	1·292
12	1·039	1·077	1·143	1·19	1·22	1·243	1·25	1·243
15	1·031	1·062	1·111	1·15	1·178	1·195	1·20	1·195
20	1·028	1·046	1·083	1·12	1·133	1·146	1·15	1·146
24	1·019	1·038	1·071	1·09	1·111	1·121	1·125	1·121

67 TABLE SHOWING THICKNESS OF DISC TO BE TAKEN FOR GRINDING LENSES (thicknesses are expressed in millimetres).

Curvature of Surface (dioptries).	Radius in Millimetres.	Diameter of Lens in Millimetres.												
		10	15	20	25	30	40	50	75	100	125	150	200	300
1	1000	..	..	..	..	0·113	0·20	0·31	0·70	1·25	1·96	2·92	5·01	11·32
1·33	750	..	..	..	0·10	0·15	0·27	0·42	0·94	1·77	2·61	3·76	6·70	15·15
1·5	666·7	..	..	..	0·11	0·17	0·30	0·47	1·06	1·88	2·97	4·23	7·54	16·10
2	500	..	..	0·100	0·16	0·23	0·40	0·63	1·41	2·51	3·92	5·29	10·10	23·03
2·5	400	..	..	0·126	0·185	0·28	0·50	0·86	1·76	3·14	4·91	7·09	12·70	29·19
3	333·3	..	..	0·14	0·23	0·34	0·59	0·935	2·12	3·77	5·91	8·91	15·35	35·66
4	250	..	0·112	0·20	0·31	0·45	0·80	1·25	2·83	5·05	7·94	11·52	20·87	50·00
5	200	..	0·14	0·25	0·390	0·56	1·00	1·57	3·55	6·35	10·02	14·60	26·80	..
6	166·7	..	0·17	0·29	0·47	0·67	1·20	1·88	4·30	7·65	12·16	17·83	33·33	..
10	100	0·126	0·28	0·50	0·78	1·13	2·02	3·18	7·30	13·40	21·94	33·80	..	..
20	50	0·251	0·57	1·01	1·59	2·30	4·17	6·70	16·94	..	..	..	..	..
40	25	0·505	1·31	2·09	3·35	5·00	10·00	25·00	..	..	..	..	..	..

Table **67** gives the thickness along the axis of lenses of various diameters and curvatures. It is to show without calculation of what thickness a disc of glass must be taken to make a plano-convex lens of given curvature. If the power of the lens is given, it must be divided by the refractivity ($\mu - 1$) of the glass in order to calculate its curvature in dioptries; then, having the curvature and diameter, the minimum thickness will be found in the Table opposite curvature and beneath the diameter.

Example I.—It is required to make a plano-convex lens of 40 mm. diameter to fit into a collar. What thickness must the disc be from which it is to be ground if its radius of curvature is 100 mm.?

In the table, opposite 100 mm. radius, and underneath 40 mm., we find 2·02 mm.; this is the minimum thickness. Now as the lens is to fit into a collar, a certain thickness at the edge is necessary to support it, say 1 mm. This must be added to the 2·02, and the result 3·02 is the thickness of disc required.

Example II.—It is required to make a double convex lens to fit into a 50 mm. collar, with surfaces having radii of curvature of 200 mm. and 100 mm. respectively. What thickness must the disc chosen possess?

Opposite 200 and below 50 we find 1·57
" 100 " 50 " 3·18

Supposing that 1·5 mm. thickness at the edge is sufficient to hold it, the total thickness of the lens after grinding must be 1·57 + 3·18 + 1·5, that is 6·25 mm., and this must be the minimum thickness of the disc.

If in the above case magnifying power had been given, this must be divided by the refractivity of the glass ($\mu - 1$), and the curvature resulting may then be split into the sum of two curvatures if a double convex lens is required, and the total thickness due to both curvatures found as above.

68 TABLE SHOWING RADIUS TO BE GROUND ON A PLANO-CONVEX LENS TO REDUCE ITS POWER TO 1 DIOPTRIE.

A plano-convex lens being given, it is required to bring its focal length to some particular value by grinding another curvature on the plano surface.

Let the radius of curvature of convex lens be called r, then the required radius x for the other face is calculated by the formula

$$x = \frac{rf(\mu - 1)}{r - f(\mu - 1)}$$

where f is the focal length to which the lens is to be brought.

If a 1-dioptrie lens is required, then $f = 1000$. The Table has been calculated with this formula for three kinds of glass. All the values are given in millimetres.

	$\mu = 1{\cdot}5$	$\mu = 1{\cdot}51$	$\mu = 1{\cdot}52$
r	x	x	x
∞	500	510	520
2000	666·6	684·5	702·7
1040	963·0	1000·8	1040
1020	980·7	1020	1060·8
1000	1000	1040·8	1083·3
800	1333·3	1406·8	1485
750	1500	1593·7	1695
700	1750	1879	2022
650	2166·6	2368	2600
600	3000	3400	3900
550	5500	7012·5	9533
500	∞	− 25500	− 13000
450	− 4500	− 3825	− 3342
400	− 2000	− 1854	− 1733
350	− 1166·6	− 1116	− 1071
300	− 750	− 728·6	− 709·1
250	− 500	− 490·4	− 481·5
200	− 333·3	− 329·0	− 325
+ 150	− 214·3	− 212·5	− 210·8
+ 100	− 125	− 124·3	− 123·8
− 100	+ 83·3	+ 83·6	+ 83·87
− 150	+ 115·38	+ 115·9	+ 116·4
− 200	+ 142·85	+ 143·6	+ 144·4

69 CONSTANTS AND DIMENSIONS OF THE EYE (Helmholtz).

	When accommodation adjusted for			
	Distant Vision.		Near Vision.	
Refractive index of the cornea	(1·3507)		(1·3507)	
Refractive index of the aqueous humour	1·3365		1·3365	
Refractive index of the vitreous humour	1·3365		1·3365	
Refractive index of the crystalline lens as a whole	1·4371		1·4371	
	mm.	inches.	mm.	inches.
Radius of curvature of the anterior surface of the cornea	7·8	0·307	7·8	0·307
Radius of curvature of the anterior surface of the crystalline lens	10·0	0·394	6·0	0·236
Radius of curvature of the posterior surface of the crystalline lens	6·0	0·236	5·5	0·217
Distance of the anterior lens surface from the vertex of the cornea	3·6	0·142	3·2	0·126
Distance of the posterior lens surface from the vertex of the cornea	7·2	0·284	7·2	0·284
Anterior focal length of the cornea	23·3	0·917	23·3	0·917
Posterior focal length of the cornea	31·1	1·224	31·1	1·224
Focal length of the lens	50·6	1·992	39·1	1·539
Distance of anterior principal point of the lens from its anterior surface.	2·1	0·0827	2·0	0·0787
Distance of posterior principal point from its posterior surface	− 1·3	− 0·0512	− 1·8	− 0·0709
Distance between principal points	0·2	0·00787	0·2	0·00787
Posterior focal length of the eye	20·7	0·815	18·7	0·736
Anterior focal length of the eye	15·5	0·610	14·0	0·551
Distance of 1st principal point from the vertex of the cornea	1·75	0·0689	1·9	0·0748
Distance of 2nd principal point from the vertex of the cornea	2·1	0·0827	2·3	0·0905
Distance of 1st nodal point from the vertex of the cornea	7·0	0·276	6·6	0·260
Distance of 2nd nodal point from the vertex of the cornea	7·3	0·288	7·0	0·276
Distance of the anterior focus from the vertex of the cornea	− 13·7	− 0·539	− 12·1	− 0·476
Distance of the posterior focus from the vertex of the cornea	22·8	0·898	21·0	0·827
	millimetres.		inches.	
Distance of the principal focus of the aphakic eye (after removal of the lens)	− 63·5 − 73·9*		2·500 2·909*	

* Measurement by Dr. Tscherning.

70 Table of Amplitude of Accommodation (from Landolt).

Age in Years.	Amplitude (Dioptries).	Distance of Near Point.	
		Millimetres.	Inches.
10	14	70	$2\frac{3}{4}$
15	12	80	$3\frac{1}{4}$
20	10	100	4
25	8·5	117	$4\frac{1}{2}$
30	7·0	140	$5\frac{1}{2}$
35	5·5	180	$7\frac{1}{8}$
40	4·5	220	$8\frac{3}{4}$
45	3·5	285	$11\frac{1}{4}$
50	2·5	400	$15\frac{3}{4}$
55	1·75	560	$22\frac{1}{2}$
60	1·0	1000	$39\frac{3}{8}$
65	0·75	..	..
70	0·0	..	..

71 Spectacle Lenses necessary for Presbyopia at different ages for an eye which in youth had normal refraction (p.p. taken as 8 inches in English practice).

Age (Years)	Power (Dioptries).		Focal Length.			
			Millimetres.		Inches.	
	If p.p.=12	If p.p.=8	p.p. = 12	p.p. = 8	p.p. = 12	p.p. = 8
45	1	0·75	1000	1333	39·37	52·49
50	2	1·25	500	800	19·68	31·50
55	3	2	333·3	500	13·12	19·68
60	4	3	260	333·3	9·84	13·12
65	4·5	3·5	222·2	286	8·75	11·26
70	5·5	4·5	181·8	222	7·16	8·75
75	6	5	166·7	200	6·56	7·87
80	7	5·5	142·9	182	5·62	6·97

REFRACTIVE INDICES OF FLUIDS COMMONLY USED IN MICROSCOPY.

Substance.	Refractive Index for D line.	Substance.	Refractive Index for D line.
Phosphorus dissolved in CS_2	1·950	Olive oil	1·470
Sulphur in carbon bisulphide	1·750	Glycerine 1 + water 1	1·397
Potassium mercury iodide	1·717	Glycerine 1 + water 1 + alcohol 1	1·394
Monobromnaphtalin	1·658	Sugar solution, 30 per cent. (aq.)	1·376
Balsam of Tolu	1·640	Potassium acetate solution (conc. aq.)	1·370
Styrax	1·630	Absolute alcohol	1·367
Oil of cinnamon	1·619	Alcohol, 40 per cent.	1·356
Cassia oil	1·607	Albumen	1·350
Aniseed oil	1·557	Sugar solution, 10 per cent. (aq.)	1·347
Canada balsam (mean)	1·535	Salt solution, 8·5 per cent. (aq.)	1·347
Clove oil	1·533	Sea water	1·343
Cedar wood oil (hardened)	1·520	Sugar solution, 5 per cent. (aq.)	1·341
Cedar wood oil	1·510	Distilled water	1·336
Castor oil	1·490	Air	1·000
Glycerine	1·473		

73 TUBE-LENGTH OF MICROSCOPES.

LENGTH IN MILLIMETRES, AND PARTS INCLUDED IN "TUBE-LENGTH" BY VARIOUS OPTICIANS. (Davis.)

Parts included in "Tube-length." See Diagram.	Various Opticians.	Tube-length in Millimetres.
a–d	Grunow, New York	203
	E. Leitz, Wetzlar	170
	Nachet et Fils, Paris	146 or 200
	Powell & Lealand, London	254
	C. Reichert, Vienna	160–180
	Spencer Lens Co., Buffalo	235 or 160
	W. Wales, New York	254
b–d	Bausch & Lomb Optical Co., Rochester, N.Y.	216 or 160
	Bézu Haiesser & Cie., Paris	220
	Klönne und Müller, Berlin	160–180 or 254
	W. & H. Siebert, Wetzlar	190
	Swift & Son, London	165–228·5
	C. Zeiss, Jena	160–250
a–g	Grundlach Optical Co., Rochester, N.Y.	254
a–g	R. Winkel, Göttingen	220
c–d	Ross & Co., London	254
c–e	R. & J. Beck, London	254
c–f	J. Green, Brooklyn, N.Y.	254
	Hartnack, Potsdam	160–180
	Vérick, Paris	160–200
	Watson & Sons, London	160–250

FIG. 53.

74 MAGNIFYING POWER OF MICROSCOPES WITH TUBES OF STANDARD LENGTH, i.e. 250 mm.; or, for English-made Microscopes, 10 inches (from Davis's "Practical Microscopy").

Eyepieces.	Objectives.													
	4″	3″	2″	1″	$\frac{3}{4}$″	$\frac{1}{2}$″	$\frac{4}{10}$″	$\frac{1}{4}$″	$\frac{1}{5}$″	$\frac{1}{6}$″	$\frac{1}{8}$″	$\frac{1}{10}$″	$\frac{1}{12}$″	$\frac{1}{16}$″
2 inch (A) . . .	12	18	25	46	50	92	130	210	275	325	400	550	650	800
$1\frac{1}{2}$ inch (B) . . .	15	23	30	54	70	110	160	250	325	390	490	650	775	980
1 inch (C) . . .	23	30	45	80	90	165	240	375	485	580	750	970	1160	1500
$\frac{3}{4}$ inch (D) . . .	30	45	60	108	140	220	320	500	650	780	980	1300	1550	1960

The Royal Microscopical Society's Standard Screw.

"Whitworth thread, i.e. a V-shaped thread, sides of thread inclined to an angle of 55° to each other, one-sixth of the V depth of the thread being rounded off at the top of the thread, and one-sixth of the thread being rounded off at the bottom of the thread. Pitch of screw, 36 to the inch; length of thread on object glass, 0·125 inch; plain fitting above thread of object glass, 0·15 inch long, to be about the size of the bottom of the male thread; length of thread of nose-piece [on the lower end of the tube of the microscope], not less than 0·125 inch; diameter of the object-glass screw at the bottom of the screw, 0·7626 inch; diameter of the nose-piece screw at the bottom of the thread, 0·8 inch."

See also *Trans. Roy. Micr. Soc.*, 1857, pp. 39–41; 1859, pp. 92–97; 1860, pp. 103–104; or *Jour. Roy. Micr. Soc.*, 1896, August. The latter paper contains a very careful and complete description of the later screw, which is almost exactly the same as the original.

75 NUMERICAL APERTURE.

A very important consideration in the performance of an objective is the quantity of light that it receives from each point in the object under examination, and this depends upon its angular aperture as seen from the point; and in the case of dry objectives it depends upon nothing else. Since the introduction of oil and homogeneous immersion it has been necessary to take into consideration the refractive index of the medium intervening between the lower lens of the objective and the cover-glass of the slide, for it is upon the value of this that the quantity of light received by a given objective from points in the object depends.

In order to make dry and immersion objectives comparable, Professor Abbe has introduced a new method of measurement which gives the capacity of objectives for receiving light as *numerical aperture.*

The numerical aperture of any objective may be defined as the *sine of half the angular aperture of an equivalent objective without immersion.* It is otherwise the sine of half the *actual* angular aperture multiplied by the refractive index of the intervening immersion medium. This may be shown as follows. See Fig. 54.

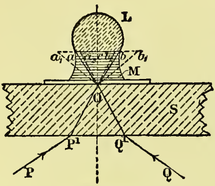

FIG. 54.

Let L represent the lower lens of an objective, S the slide and cover-glass, M the immersion medium, P P¹ O Q¹ Q a pencil of rays converging from the condenser to the point O in the object under examination, and on the optic axis; it undergoes refraction at the lower surface of the slide S as shown. We have taken the refractive index of the immersion medium and the convergence of the pencil, so that after refraction into M the pencil just fills up the aperture of the lens. If M had the same index as glass, the rays composing the pencil would have proceeded unrefracted, and the pencil would have spread over an area represented by a_2 b_2; if the medium M were replaced by air, the rays would be

refracted outwards, and the pencil spread over an area represented by a_1 b_1, a large amount of light being lost in the latter case. Then for a dry-objective to receive the same amount of light as that in the figure, its semi-angular aperture would have to be $c\,o\,a_1$ as compared with $c\,o\,a$.

Now $o\,b_1$ and $o\,a_1$ are parallel respectively to P P[1] Q Q[1], and by the law of successive refraction it is easily seen that $\frac{\sin\ c\,o\,a_1}{\sin\ c\,o\,a} = \mu_m$, the refractive index of the medium M, or in other words the sine of the semi-angular aperture of the equivalent dry objective is equal to the size of the actual semi-angular diameter multiplied by the refractive index of the immersion medium (which in the case of air is unity).

Upon the numerical aperture depend also the resolving, penetrating and illuminating power. The *Resolving Power* is the ability a given objective possesses in rendering detail clear. It varies directly as the numerical aperture, *cæteribus paribus*. The *Penetrating Power*, or "depth of focus," is the ability to bring into focus more than one section or plane of the object at once. It varies inversely as the numerical aperture, and directly as the square of the focal length for the same relative aperture, or inversely as the focal length for the same absolute aperture. The *Illuminating Power* varies as the square of the numerical aperture.

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Table of Numerical Aperture.

Objective.	Angular Aperture 2α.	Index of Refraction of Immersion Medium μ.	Sine of half Angular Aperture sine α.	Numerical Aperture μ sin α.
1" (dry)	20°	1	0·1736	0·1736
1" "	40°	1	0·3420	0·3420
$\frac{1}{2}$" "	42°	1	0·3583	0·3583
$1\frac{1}{2}$" "	100°	1	0·7660	0·7660
$\frac{1}{4}$" "	75°	1	0·6087	0·6087
$\frac{1}{4}$" "	136°	1	0·9272	0·9272
$\frac{1}{8}$" "	115°	1	0·8434	0·8434
$\frac{1}{8}$" "	163°	1	0·9890	0·9890
$\frac{1}{12}$" Water immersion.	96° 12′	1·33	0·7443	0·9899
$\frac{1}{12}$" Homogeneous immersion.	110° 38′	1·52	0·8223	1·2500
$\frac{1}{12}$" Homogeneous immersion.	134° 10′	1·52	0·9210	1·4000

77 RESOLVING POWER OF MICROSCOPE OBJECTIVES (from Dippel).

Numerical Aperture N A = μ sin α.	Angle of Aperture (2α).			Theoretical Limit of Resolving Power.			
				Direct Light.		Oblique Light.	
	Dry $\mu = 1$.	Water Immersion $\mu = 1{\cdot}33$.	Homogeneous Immersion $\mu = 1{\cdot}52$.	Width between in Mikrons.	Number visible in 10 Mikrons.	Width between in Mikrons.	Number visible in 10 Mikrons.
0·15	17°	..	..	1·70	6	..	..
0·20	23°	..	..	1·40	7	..	..
0·25	29°	..	..	1·00	10	..	..
0·30	35°	..	..	0·90	11	..	..
0·35	41°	..	..	0·80	12	..	..
0·40	47°	..	..	0·74	13	0·68	14
0·45	53°	..	..	0·70	14	0·60	16
0·50	60°	..	..	0·65	15	0·55	18
0·55	66°	..	..	0·60	16	0·50	20
0·60	74°	..	..	0·58	17	0·45	22
0·65	82°	..	..	0·55	18	0·42	24
0·70	90°	..	..	0·53	19	0·39	25
0·75	97°	..	..	0·50	20	0·36	28
0·80	106°	..	..	0·48	21	0·34	29
0·85	116°	..	..	0·46	22	0·32	30
0·90	128°	85°	..	0·44	23	0·30	33
0·95	144°	91°	..	0·42	24	0·29	34
1·00	180°	97°	82°	0·41	25	0·27	36
1·05	..	104°	86°	0·39	26	0·26	38
1·10	..	112°	92°	0·38	27	0·25	40
1·15	..	119°	98°	0·36	28	0·24	41
1·20	..	128°	104°	0·35	28	0·23	43
1·25	..	140°	113°	0·34	29	0·22	45
1·30	..	156°	120°	0·335	30	0·21	47
1·35	..	..	128°	0·32	31	0·20	50
1·40	..	..	138°	0·315	31–32	0·195	51
1·45	..	..	145°	0·30	33	0·19	52

This table is calculated upon the supposition that the mean wave-length of white light amounts to 55 microcentimetres (see Table **15**), and the value of $\alpha = \mu \sin W = 0{\cdot}342$; where μ is the refractive index, and W the inclination of the light entering the microscope to the optic axis. From the equation $a = \frac{\lambda}{e} - \alpha$, is obtained the smallest numerical aperture a, by which a system of *parallel lines* with given width e between them would be visible. N.B.—A *Mikron* = 10^{-6} metre. See Table **11**, p. 16.

78 Table showing **DEPTH OF VISION** or greatest distance between two sections visible or in focus at the same time with various degrees of amplification (Abbe).

Amplification with Numerical Aperture = ·5.	*a.* Diameter of Field.	*b.* Accommodation Depth.	*c.* Focal Depth.	*d.* Depth of Vision $b + c$.	Ratio of *a* to *d.* $\frac{a}{d}$.
	mm.	mm.	mm.	mm.	
10	25·0	2·08	0·073	2·153	11·6 to 1
30	8·3	0·23	0·024	0·254	32·7 „
100	2·5	0·02	0·0073	0·0273	91·6 „
300	0·83	0·0023	0·0024	0·0047	176·6 „
1000	0·25	0·00021	0·00073	0·00094	266 „
3000	0·083	0·00002	0·00024	0·00026	319 „

79 PENETRATING POWER AND DEPTH OF FOCUS. **That is the distance between the extreme planes in focus at the same time with various objectives used with an A eye-piece (Davis).**

Objective.	Air Angle or Angular Aperture $2\alpha°$.	N.A. Numerical Aperture $= \mu \sin \alpha$.	Penetrating Power $\frac{1}{\text{N.A.}}$	Depth of Focus in Mikrons.	Depth of Accomodation in Mikrons.	Total Depth of Focus in Mikrons.
4″	8°	·07	14·30	522	2080	2602
4″	16°	·14	7·19	262	2080	2342
$1\frac{1}{2}$″	16°	·14	7·19	86	230	316
$1\frac{1}{2}$	20°	·17	5·75	69	230	299
$1\frac{1}{2}$	24°	·21	4·81	57	230	287
$\frac{1}{2}$	40°	·34	2·92	10·6	20	30·6
$\frac{1}{2}$	70°	·57	1·74	6·3	20	26·3
$\frac{1}{2}$	110°	·82	1·22	4·4	20	24·4
$\frac{1}{6}$	74°	·60	1·66	1·99	2·3	4·29
$\frac{1}{6}$	100°	·76	1·31	1·57	2·3	3·87
$\frac{1}{6}$	..	1·20	·83	0·99	2·3	3·29
$\frac{1}{12}$	110°	·83	1·20	0·72	0·58	1·30
$\frac{1}{12}$	144°	·97	1·02	0·61	0·58	1·19
$\frac{1}{12}$	..	1·10	·91	0·54	0·58	1·12
$\frac{1}{20}$	160°	·98	1·02	0·37	0·21	0·58
$\frac{1}{20}$	..	1·10	·91	0·33	0·21	0·54

A photograph is generally considered distinct when the images of the various points in the object do not subtend more than an angle of 1 minute at the eye, when it is held at the distance of distinct vision, say, 10 inches. Under these conditions the actual diameters of these images on the photographic plate is less than $\frac{1}{300}$th of 1 inch. Now in Fig. 55 let P O Q represent a pencil of rays emanating from a point, but converging to O after traversing a lens, and let the cross-sections B C, A D, of the pencil be supposed equal to $\frac{1}{300}$ inch in diameter; then from what has been said a plate placed anywhere between A and B will receive an apparently sharp image of the point. The distance A B is called the "depth of focus." It is also clear from the figure that the depth of focus is smaller, the greater the aperture ratio; that is to say, the greater the angle included by the cone of rays.

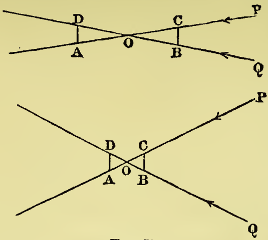

Fig. 55.

The following tables give the distances beyond which all objects are in focus, because of "depth of focus," when the lens is adjusted for infinity.

MINIMUM CAMERA-DISTANCES FOR SHARP PICTURES WITH GIVEN APERTURE-RATIOS (from Miethe).

Aperture-Ratio.	Focal Length.										
	50 mm.	75 mm.	100 mm.	125 mm.	150 mm.	175 mm.	200 mm.	250 mm.	300 mm.	350 mm.	400 mm.
$f/5$	2·5	5·6	10·0	15·5	22·5	30·7	40·0	62·0	90·0	122·0	160·0
$f/10$	1·3	2·8	5·0	8·0	11·0	15·0	20·0	31·0	45·0	61·0	80·0
$f/15$	0·8	1·9	3·3	5·1	8·0	10·0	13·0	21·0	30·0	41·0	53·0
$f/20$	0·7	1·4	2·5	4·0	5·5	7·5	10·0	15·0	22·0	31·0	40·0
$f/25$	0·5	1·1	2·0	3·0	4·5	6·0	8·0	12·0	18·0	24·0	32·0
$f/30$	0·4	0·9	1·6	2·5	4·0	5·0	6·5	10·0	15·0	20·0	26·0
$f/40$	0·3	0·7	1·2	2·0	2·5	3·5	5·0	7·5	11·0	15·5	20·0
$f/50$	0·2	0·6	1·0	1·5	2·5	3·0	4·0	6·0	9·0	12·0	16·0

This table gives the distances in metres at which the depth-aberration is annulled for any given stop for lenses of different focal lengths. It is computed on the assumption that an image may be considered sharp when the image of a point in the object does not exceed 0·1 mm. in diameter.

Aperture-Ratio.	Focal Length.												
	4″	4½″	5″	5½″	6″	6½″	7″	7½″	8″	8½″	9″	9½″	10″
	Distance in feet at which depth aberration is annulled.												
f/4	33·7	42·5	52·5	63·6	75·5	88·7	102·6	117·8	133·9	151·0	169·3	188 8	209·1
f/5·6	24·1	30·5	37·6	45·6	54·1	63·5	73·4	84·3	96·0	108·2	121·2	135·0	149·6
f/6	22·6	28·5	35·1	42·6	50·5	59·3	68·6	78·8	89·6	101·0	113·2	126·0	139·6
f/7	19·4	24·5	30·3	36·6	43·3	50·8	58·9	67·6	76·9	86·7	97·2	108·2	119·8
f/8	17·0	21·5	26·5	32·1	38	44·6	51·6	59·2	67·4	76·0	85·2	94·8	104·8
f/10	13·7	17·2	21·3	26·0	30·5	35·8	41·0	47·5	54·0	60·9	67·3	76·0	84·2
f/11	12·4	15·7	19·3	23·4	27·8	32·5	37·7	43·2	49·2	55·4	62·1	69·1	76·6
f/15	9·2	11·6	14·3	17·4	20·5	24·0	27·8	31·9	36·2	40·8	45·7	50·9	56·4
f/16	8·7	10·9	13·4	16·2	19·2	22·6	26·1	29·9	34·0	38·3	42·9	47·8	52·9
f/20	7·0	8·8	10·8	13·1	15·5	18·2	21·0	24·0	27·3	30·8	34·5	38·4	42·4
f/22	6·4	8·0	9·9	11·9	14·1	16·6	19·2	21·9	25·0	28·1	31·4	35·0	38·7
f/32	4·5	5·7	6·9	8·3	9·9	11·5	13·4	15·3	17·3	19·5	21·9	24·3	26·8
f/44	3·4	4·2	5·2	6·2	7·3	8·6	9·9	11·3	12·8	14·4	16·1	17·9	19·8
f/64	2·4	3·0	3·8	4·4	5·2	5·6	7·0	8·0	9·0	10·1	11·1	12·6	13·8

This table is calculated from the formula

$$d = f\left(\frac{100f}{b} + 1\right)$$

where d = distance in feet at which depth aberration is annulled, measured from the lens.

f = focal length of the camera lens.

b = ratio of focal length to the diameter of the stop, the aperture ratio being $\frac{f}{b}$ according to the usual nomenclature.

81 TABLE OF SIZES OF THE DIFFRACTION DISC (from Miethe).

Linear Aperture.	Diffraction Disc Ratio.	Linear Aperture.	Diffraction Disc Ratio.
mm. 100	0·0000139	mm. 5	0·0002788
50	0·0000280	4	0·0003482
20	0·0000697	3	0·0004645
10	0·0001390	2	0·0006968
8	0·0001742	1	0·0013931
6	0·0002323		

In column 1 are set down values of the diameters of the aperture of the objective. The figures in column 2 give the ratio which the diameter of the corresponding diffraction-disc (see in Table **57** Aberrations due to Aperture) bears to the focal length of the lens.

82 DISTANCE OF OPTICAL LANTERN FROM SCREEN TO PRODUCE REQUIRED SIZE OF DISC.

Focal Length of Lens.	Size of Screen.						
	6	9	12	15	20	24	30
	Distance of Lantern from Screen.						
inches.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
2	4	6	8	10	$13\frac{1}{3}$	16	20
3	6	9	12	15	20	24	30
4	8	12	16	20	26	32	40
5	10	15	20	25	$33\frac{1}{3}$	40	50
6	12	18	24	30	40	48	60
8	16	24	32	40	$53\frac{1}{3}$	64	80
9	18	27	36	45	60	72	90
12	24	36	48	60	80	96	120

83 REDUCTION TABLE FOR FIGURES AND HEADS.

Ratio of the Size of the Image to that of the Object.	The actual Size of the Image of a Man.	The actual Size of the Image of a Head.	Ratio of the Size of the Image to that of the Object.	The actual Size of the Image of a Man.	The actual Size of the Image of a Head.
	mm.	mm.		mm.	mm.
$\frac{1}{1}$	1750	210	$\frac{1}{35}$	50	6
$\frac{1}{2}$	875	105	$\frac{1}{40}$	44	5·25
$\frac{1}{3}$	583	70	$\frac{1}{45}$	39	4·75
$\frac{1}{4}$	437	52	$\frac{1}{50}$	35	4·25
$\frac{1}{5}$	350	42	$\frac{1}{60}$	29	3·5
$\frac{1}{6}$	292	35	$\frac{1}{70}$	25	3
$\frac{1}{7}$	250	30	$\frac{1}{80}$	22	2·5
$\frac{1}{8}$	219	26	$\frac{1}{90}$	19	2·33
$\frac{1}{9}$	194	23	$\frac{1}{100}$	18	2·2
$\frac{1}{10}$	175	21	$\frac{1}{120}$	15	1·75
$\frac{1}{15}$	117	14	$\frac{1}{140}$	13	1·5
$\frac{1}{20}$	88	11	$\frac{1}{160}$	11	1·33
$\frac{1}{25}$	70	8	$\frac{1}{180}$	10	1·2
$\frac{1}{30}$	58	7	$\frac{1}{200}$	9	1

ENLARGEMENT AND REDUCTION TABLE.

Focal Length.	Ratio of Size of Image to Size of Object.																									
	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{50}$	$\frac{1}{60}$	$\frac{1}{70}$	$\frac{1}{80}$	$\frac{1}{90}$	$\frac{1}{100}$	$\frac{1}{120}$	$\frac{1}{140}$	$\frac{1}{160}$	$\frac{1}{180}$	$\frac{1}{200}$
	Distances of Object and Image from the Lens.																									
1	2 2	3 1·5	4 1·3	5 1·3	6 1·2	7 1·2	8 1·1	9 1·1	10 1·1	11 1·1	15 1·1	21 1·1	26 1·0	31 1·0	41 1·0	51 1·0	51 1·0	71 1·0	81 1·0	91 1·0	101 1·0	121 1·0	141 1·0	161 1·0	181 1·0	201 1·0
1·5	3 3	4·5 2·3	6 2	7·5 1·9	9 1·8	10·5 1·8	12 1·7	13·5 1·7	15 1·7	16·5 1·7	24 1·6	31·5 1·5	39 1·6	46·5 1·5	61·5 1·5	76·5 1·5	91·5 1·5	105·5 1·5	121·5 1·5	135·5 1·5	151·5 1·5	181·5 1·5	211·5 1·5	241·5 1·5	271·5 1·5	301·5 1·5
2	4 4	5 3	8 2·7	10 2·5	12 2·4	14 2·3	15 2·3	18 2·3	20 2·2	22 2·2	32 2·1	42 2·1	52 2·1	62 2·1	82 2·1	102 2·0	122 2·0	142 2·0	162 2·0	182 2·0	202 2·0	242 2·0	282 2·0	322 2·0	362 2·0	402 2·0
2·5	5 5	7·5 3·8	10·0 3·3	12·5 3·1	15·0 3·0	17·5 2·9	20·0 2·9	22·5 2·8	25·0 2·8	27·5 2·8	40 2·7	52·5 2·6	55·0 2·6	77·5 2·6	102·5 2·6	127·5 2·6	152·5 2·5	177·5 2·5	202·5 2·5	227·5 2·5	252·5 2·5	302·5 2·5	352·5 2·5	402·5 2·5	452·5 2·5	502·5 2·5
3	6 6	9 4·5	12 4·0	15 3·8	18 3·6	21 2·5	24 3·4	27 3·4	30 3·3	33 3·3	48 3·2	63 3·2	78 3·1	93 3·1	123 3·1	153 3·1	183 3·1	213 3·0	243 3·0	273 3·0	303 3·0	363 3·0	423 3·0	483 3·0	543 3·0	603 3·0
3·5	7 7	10·5 5·3	14 4·7	17·5 4·4	21 4·2	24·5 4·1	28 4·0	31·5 3·9	35 3·9	38·5 3·9	56 3·7	73·5 3·7	91 3·6	108·5 3·6	143·5 3·6	178·5 3·6	213·5 3·6	248·5 3·6	283·5 3·5	318·5 3·5	353·5 3·5	423·5 3·5	493·5 3·5	563·5 3·5	633·5 3·5	703·5 3·5
4	8 8	12 6	16 5·3	20 5	24 4·8	28 4·7	32 4·6	35 4·5	40 4·4	44 4·4	64 4·3	84 4·2	104 4·2	124 4·1	164 4·1	204 4·1	244 4·1	280 4·1	324 4·1	364 4·0	404 4·0	484 4·0	564 4·0	644 4·0	720 4·0	804 4·0
4·5	9 9	13·5 6·8	18 5	22·5 5·6	27 5·4	31·5 5·3	36 5·1	40·5 5·1	45 5	49·5 5	72 4·8	94·5 4·7	117 4·7	139·5 4·7	184·5 4·6	229·5 4·5	274·5 4·5	319·5 4·6	354·5 4·6	409·5 4·6	454·5 4·5	544·5 4·5	634·5 4·5	724·5 4·5	814·5 4·5	904·5 4·5
5	10 10	15 7·5	20 5·7	25 5·3	30 6·0	35 5·8	40 5·7	45 5·6	50 5·5	55 5·5	80 5·3	105 5·3	130 5·2	155 5·2	205 5·1	255 5·1	305 5·1	355 5·1	405 5·1	455 5·1	505 5·1	605 5·0	705 5·0	805 5·0	905 5·0	1005 5·0
6	12 12	18 9	24 8	30 7·5	35 7·2	42 7·0	48 6·9	54 6·8	60 5·6	66 6·6	96 6·4	126 6·3	155 5·2	186 5·2	246 6·2	306 6·1	366 6·1	425 6·1	485 6·1	545 6·1	606 6·1	725 6·1	845 6·0	965 6·0	1085 6·0	1206 6·0
7	14 14	21 10·5	28 9·3	35 8·7	42 8·4	49 8·2	56 8·0	63 7·9	70 7·7	77 7·7	112 7·5	147 7·4	182 7·3	217 7·2	287 7·2	357 7·1	427 7·1	497 7·1	557 7·1	637 7·1	707 7·1	847 7·1	987 7·1	1127 7·0	1267 7·0	1407 7·0

(120)

This Table may be used for Inches, or for any other unit of length. If the focal length of the lens is given in inches, then the figures in the Table will give *in inches, also*, the distances of object and image, which are conjugate one to the other. For *Example* :—Given an 8-inch lens it is desired to find the positions of object and image.

ENLARGEMENT AND REDUCTION TABLE—*continued.*

Focal Length.	Ratio of Size of Image to Size of Object.																									
	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{20}$	$\frac{1}{25}$	$\frac{1}{30}$	$\frac{1}{40}$	$\frac{1}{50}$	$\frac{1}{60}$	$\frac{1}{70}$	$\frac{1}{80}$	$\frac{1}{90}$	$\frac{1}{100}$	$\frac{1}{120}$	$\frac{1}{140}$	$\frac{1}{160}$	$\frac{1}{180}$	$\frac{1}{200}$
	Distances of Object and Image from the Lens.																									
8	16 16	24 12	32 10·7	40 10	48 9·6	65 9·3	64 9·1	72 9	80 8·8	88 8·8	128 8·5	158 8·4	208 8·3	248 8·3	328 8·2	408 8·2	488 8·1	668 8·1	648 8·1	728 8·1	808 8·1	958 8·1	1128 8·1	1288 8·1	1448 8·0	1608 8·0
9	18 18	27 13·6	36 12	45 11·2	64 10·8	63 10·5	72 10·3	81 10·1	90 9·9	99 9·9	144 9·6	189 9·5	234 9·4	279 9·3	369 9·2	469 9·2	549 9·2	639 9·1	729 9·1	819 9·1	909 9·1	1089 9·1	1259 9·1	1449 9·1	1629 9·1	1809 9·0
10	20 20	30 16	40 13·3	60 12·6	60 12	70 11·7	80 11·4	90 11·3	100 11	110 11	160 10·7	210 10·5	260 10·4	310 10·3	410 10·3	610 10·2	610 10·2	710 10·1	810 10·1	910 10·1	1010 10·1	1210 10·1	1410 10·1	1610 10·1	1810 10·1	2010 10·1
12	24 24	36 18	48 16	60 16	72 14·4	84 14	95 13·7	108 13·6	120 13·3	132 13·2	192 12·8	252 12·6	312 12·5	372 12·4	492 12·3	612 12·2	732 12·2	852 12·2	972 12·2	1092 12·1	1212 12·1	1452 12·1	1592 12·1	1932 12·1	2172 12·1	2412 12·1
14	28 28	42 21	55 18·7	70 17·6	84 16·8	98 15·3	112 15	126 16·8	140 16·5	154 15·4	224 15·0	294 14·7	364 14·6	434 14·6	574 14·4	714 14·3	854 14·2	994 14·2	1134 14·2	1274 14·2	1414 14·1	1694 14·1	1974 14·1	2254 14·1	2534 14·1	2812 14·1
16	32 32	48 24	54 21·3	80 20	96 19·2	112 18·7	128 18·3	144 18·0	160 17·8	176 17·6	266 17	335 16·8	416 16·5	495 16·5	556 16·4	815 16·3	976 16·3	1136 15·2	1296 16·2	1456 15·2	1616 15·2	1935 16·1	2256 16·1	2576 15·1	2896 15·1	3216 15·1
18	35 36	54 27	72 24	90 22·6	108 21·6	125 21	144 20·6	152 20·3	180 20	198 19·8	288 19·2	378 18·9	468 18·7	558 18·6	738 18·6	918 18·3	1098 18·3	1278 18·3	1458 18·2	1638 18·2	1818 18·2	2178 18·2	2538 18·1	2898 18·1	3258 18·1	3618 18·1
20	40 40	60 30	80 26·7	100 26	120 24	140 23·3	160 22·9	180 22·6	200 22·2	220 22	320 21·3	420 21	620 20·8	620 20·7	820 20·6	1020 20·4	1220 20·3	1420 20·3	1620 20·3	1820 20·2	2020 20·2	2420 20·2	2820 20·1	3220 20·1	3520 20·1	4020 20·1
25	50 50	76 37·6	100 33·3	126 31·3	160 30	176 29·2	200 28·6	226 28·1	250 27·8	276 27·6	400 26·7	626 25·3	650 25	775 25·8	1025 25·6	1275 26·6	1525 25·4	1776 26·4	2025 25·3	2275 25·3	2525 25·3	3026 25·2	3525 25·2	4026 25·2	4525 25·1	5025 25·1
30	60 60	90 46	120 40	150 37·6	180 36	210 35	240 34·3	270 33·8	300 33·3	330 33	480 32	630 31·6	780 31·2	930 31	1230 30·8	1630 30·6	1830 30·6	2130 30·4	2430 30·4	2730 30·3	3030 30·3	3630 30·3	4230 30·2	4830 30·2	5430 30·2	5030 30·2

to produce a *reduction* of $\frac{1}{4}$ in size. Looking down the column headed with the ratio $\frac{1}{4}$ we find opposite the focal length of 8 inches, that the object must be 40 inches in front, and the image 10 inches behind. If, on the other hand, with the same lens we wished to prodnce an enlargement of 4 times, i.e. in the ratio of $\frac{4}{1}$; the same figures must be taken in the inverse way; the image being produced at 40 inches behind, when the object is placed 10 inches in front of the lens.

85 TABLE OF VIEW-ANGLES FOR CAMERA (after Woodman).

Divide the Width of Plate by Focal Length (equivalent) of Lens.

Quotient.	View Angle.	Quotient.	View Angle.	Quotient.	View Angle.
·282	16°	·748	41°	1·3	66°
·3	17°	·768	42°	1·32	67°
·317	18°	·788	43°	1·36	68°
·335	19°	·808	44°	1·375	69°
·353	20°	·828	45°	1·4	70°
·37	21°	·849	46°	1·427	71°
·389	22°	·87	47°	1·45	72°
·407	23°	·89	48°	1·48	73°
·425	24°	·911	49°	1·5	74°
·443	25°	·933	50°	1·53	75°
·462	26°	·954	51°	1·56	76°
·48	27°	·975	52°	1·59	77°
·5	28°	1·0	53°	1·62	78°
·517	29°	1·02	54°	1·649	79°
·536	30°	1·041	55°	1·678	80°
·555	31°	1·063	56°	1·7	81°
·573	32°	1·086	57°	1·739	82°
·592	33°	1·108	58°	1·769	83°
·611	34°	1·132	59°	1·8	84°
·631	35°	1·155	60°	1·833	85°
·65	36°	1·178	61°	1·865	86°
·67	37°	1·2	62°	1·898	87°
·689	38°	1·225	63°	1·931	88°
·708	39°	1·25	64°	1·965	89°
·728	40°	1·274	65°	2·0	90°

Example.—The following are the principal sizes of photographic plates.

$3\frac{1}{4} \times 4\frac{1}{4}$	Quarter Plate.
$6\frac{1}{2} \times 4\frac{3}{4}$	Half Plate.
$6\frac{1}{2} \times 8\frac{1}{2}$	Whole Plate.
5×4	
$7\frac{1}{2} \times 5$	
10×8	
12×10	
15×12	

Given a lens of 9 *inches focal length, find the view angle included by it on a half plate.*

The greatest width of a half plate is $6\frac{1}{2}$ inches. Hence the quotient is $6\frac{1}{2} \div 9 = 0{\cdot}722$. Comparing this with the table, we see that this value is intermediate between ·708 and ·728. Hence the view angle will be between 39° and 40°.

86 Table of Sensibility of the Normal Eye to Light from different parts of the Spectrum (compiled from Abney).

Frauenhofer Line, and Colour.	λ (see Table **15.**)	Outside Yellow Spot.	Yellow Spot.	Fovea Centralis.
Full red B . .	686·7	1·5	3	3
Orange red C . .	656·3	10·5	17·6	20·6
Mid orange . .	624·2	35	65	77
Full yellow D . .	589·6	71·0	99	100
Mid yellow . .	585·0	79	100	98
Primrose . . .	572·0	85	97	90
Mid green . .	548·1	72·5	75	66
Green E . . .	527·0	52	50	40
Peacock . . .	504·3	33·5	24	18
Blue F . . .	486·1	17·5	6·3	4·7
Deep Blue G . .	430·8	2·0	0·50	0
Violet h . . .	410·18	0·5	0·14	0

87 Relative Sensations produced in the Eye by equal Quantities of Energy in different parts of the Spectrum (from Palaz).

Colour.	λ See Table **15.**	Relative Luminous Sensation.
Dull red	759	1
Red	656	1200
Orange	600	14000
Yellow	580	28000
Green	530	100000
Blue	470	62000
Violet	400	1600

88 PHOTOMETRIC UNITS.

The *bougie decimale* is $\frac{1}{20}$ Violle unit; it therefore equals 0·925 British candles, or 0·945 Hefner units.

The *unit of illumination of a surface* is the amount of illumination of that surface produced by *one bougie decimale at the distance of one metre.* British engineers often take as a unit of illumination the illumination produced by *one candle at the distance of one foot.* The Geneva Congress of 1896 adopted, however, the former unit, together with the following:—

Quantity to be Expressed.	Name of Corresponding Unit.
Intrinsic Light Intrinsic Brilliancy Candle-power	Bougie decimale.
Illumination of Surface	1 *Lux* = 1 Bougie decimale at distance of 1 metre.
Quantity of Light Total light in a pencil Luminous Flux	1 *Lumen* = the flux due to 1 bougie decimale within a solid angle equal to unity. Hence the total flux all round from a light of 1 bougie decimale = 4 π lumens. 1 Lumen spread over 1 square metre gives an illumination of 1 lux.
Brilliancy / Luminosity of flame Specific luminosity	1 Bougie decimale per square centimetre.
Quantity of Lighting	1 lumen-hour.

89 COMPARISON OF PHOTOMETRIC STANDARDS.

	Violle Units.	Carcel Lamp.	Star Candles.	German Candles.	English Candles.	Hefner Lamp.
Violle unit	1·000	2·08	16·1	16·4	18·5	18·9
Carcel	0·481	1·00	7·75	7·89	8·91	9·08
Star candle	0·062	0·130	1·00	1·02	1·15	1·17
German candle	0·061	0·127	0·984	1·00	1·13	1·15
English candle	0·054	0·112	0·870	0·886	1·00	1·02
Hefner Lamp.	0·053	0·114	0·853	0·869	0·98	1·00

Electric Arc = from 110 to 160 candles per square millimetre of crater surface; according to Blondel, 158 bougie decimales per sq. mm.; according to Petavel, 147 bougie decimales per sq. mm.

Harcourt's Pentane Lamp, burning pure *pentane*, under standard conditions, is constructed in several sizes from 1 to 10 British candles.

The Hefner Lamp, burning *amyl acetate*, with a rather reddish flame, is adopted in Germany as a standard.

Blondel has proposed a whiter standard lamp of equal total power, burning a mixture of 84 parts by volume of *absolute alcohol* with 16 parts of pure crystallizable *benzol*.

Flame standards may vary as much as 4 per cent. with humidity of air.

90 DIFFUSE REFLEXION (Sumpner).

The Reflective power of a surface of given area is the ratio of the quantity of light reflected to the quantity incident upon it.

The Brightness of a diffusing surface is its candle-power per unit area in a direction normal to its surface.

The Illumination of a surface is the quantity of light falling on it per second per unit of area.

If η is the reflective power of a surface, B the brightness, I the illumination, then $\pi B = \eta I$.

In an enclosed space containing sources of light, let I′ be the average illumination of the walls, I the illumination of the walls due to the direct light of the sources alone.

Then $I' = \dfrac{1}{1-\eta} I$.

DIFFUSE REFLECTIVE POWER OF VARIOUS MATERIALS.

Material.	η (per cent.)	$\dfrac{1}{1-\eta}$
White blotting paper	82	5·5
„ cartridge paper	80	5·0
Tracing cloth	35	1·54
„ paper	22	1·28
Ordinary foolscap	70	3·33
Newspapers	50–70	2–3·33
Tissue paper (one thickness)	40	1·66
„ „ (two thicknesses)	55	2·22
Yellow wall-paper	40	1·66
Blue paper	25	1·33
Dark brown paper	13	1·15
Deep chocolate paper	4	1·03
Plane deal (clean)	40–50	1·66–2
„ „ (dirty)	20	1·25
Yellow cardboard	30	1·43
Parchment (one thickness)	22	1·28
„ (two thicknesses)	35	1·54
Yellow painted wall (clean)	40	1·66
„ „ „ (dirty)	20	2·22
Black cloth	1·2	1·01
„ velvet	0·4	1·004

In the above table η is the reflective power expressed as percentage of incident light; $\dfrac{1}{1-\eta}$ is the coefficient by which the illumination of walls of a room by direct light from the source of illumination must be multi lied in order to obtain the average illumination.

91 Reflective Power in Percentage of Incident Light (Hagen & Rubens).

	Blue $\lambda=450$	Green $\lambda=500$	Yellow $\lambda=550$	Orange $\lambda=600$	Red $\lambda=700$
A. CLEAN METALS.	%	%	%	%	%
Silver	90·6	91·8	92·5	93·0	94·6
Platinum	55·8	58·4	61·1	64·2	70·1
Nickel	58·5	60·8	62·6	64·9	69·8
Steel, hardened	58·6	59·6	59·4	60·0	60·7
Steel, unhardened . . .	56·3	55·2	55·1	56·0	59·3
Gold	36·8	47·3	74·7	85·6	92·3
Copper	48·8	53·3	59·5	83·5	90·7
B. SPECULUM METALS.					
Rosse's alloy—					
68·2 % Cu + 31·8 % Sn . .	62·9	63·2	64·0	64·3	67·3
Brashear's alloy—					
68·2 % Cu + 31·8 % Sn . .	61·9	63·3	64·0	64·4	68·5
Schröder's alloy No. 6—					
60 % Cu + 30 % Sn + 10 % Ag	61·5	62·5	63·6	65·2	68·6
Ludwig Mach's alloys—					
No. 1 (2 pts. Al + 1 pt. Mg) .	83·4	83·3	82·7	83	83·3
No. 7 (1 pt. Al + 1·5 pts. Mg)	83·4	82·5	82·1	83·8	84·4
No. 12 (1 pt. Al + 2·75 pts. Mg)	83·4	84·5	83·8	84·5	83·8
C. GLASS MIRRORS.					
Backed with silver . . .	82·5	84·1	85·4	85·3	87·1
Backed with mercury amalgam .	72·8	70·9	71·2	69·9	72·8

Sheet glass reflects, at perpendicular incidence, about 8·7 per cent., and transmits about 91·3 per cent. of the light that falls upon it. A transparent substance having refractive index μ reflects the fraction of it represented by $(\mu-1)^2\ (\mu+1)^2$.

	Name and Tint.	Density.	Hardness.	Crystal System.	Twin Colours seen in Dichroïscope.
RED STONES.	Ruby; pink to deep red: best tint pigeon's blood.	3·95	8–9	II	Darker and more purple red; paler and redder tint.
	Spinel; pink to deep crimson	3·5	8	I	Both alike.
	Rubellite (Pink Tourmaline); rose or rose pink.	3·15	7–8	III	Pink or red; white or pale red.
	Pink Topaz; pale pink	3·5	8	IV	Pale rose; yellow.
	Red Diamond; pink or ruby red	3·52	10	I	Both alike.
	Garnet (Almandine; "Cape Ruby"); dark red, brownish red, or purplish red.	3·6 to 4·2	7	I	Both alike.
	Jacinth; dark sherry	4·5	7–8	II	Red-brown; greenish.
	Rose Quartz; pale rose	2·66	7	III	
ORANGE OR BROWN.	Jacinth; brown red or cinnamon	4·5	7–8	II	Red-brown; brownish green.
	Garnet; brown red	3·6 to 4·2	7	I	Both alike.
	Diamoud; cinnamon or brown	3·52	10	I	Both alike.
	Tourmaline; brown or deep orange	3·15	7–8	III	Brown; straw.
	Epidote; dark brown	3·3	6–7	V	Dark brown; pale brown.
YELLOW.	Topaz; full yellow to pale yellow	3·5	8	IV	Deep yellow; pale yellow.
	Cairngorm (Yellow Quartz)	2·66	7	III	
	Yellow Sapphire; bright yellow	3·95	9	II	Yellow; greenish.
	Yellow Beryl; lemon yellow	2·71	7–8	III	
	Chrysoberyl; lemon yellow	3·7	8–9	IV	Straw yellow; greenish yellow.
	Yellow Diamond; canary	3·52	10	I	Both alike.
	Amber; pale yellow to orange	1·03	Soft	O	Both alike.

COLOURS, DENSITY AND HARDNESS OF GEMS—*continued.*

	Name and Tint.	Density.	Hardness.	Crystal System.	Twin Colours seen in Dichroïscope.
GREEN.	Emerald; emerald green	2·71	8	III	Blue green; yellow green.
	Hiddenite; fuller green than emeralds	3·15	6½–7	V	Green; yellow green.
	Green Garnet; gooseberry green	3·5	8	I	Both alike.
	Green Sapphire; dull green	3·95	9	II	Blue green; yellowish green.
	Olivine (Peridot, Chrysolite); sage green	3·37	6–7	IV	Pea green; dull yellow.
	Tourmaline; darker green than emerald.	3·15	7–8	III	Blue green; yellow or straw.
	Moroxite (Apatite); sage green	3·26	5	III	
	Green Diamond; pale grass	3·52	10	I	Both alike.
	Chrysoprase; turbid green	2·66	7	III	
	Alexandrite; dull emerald by daylight; mulberry red by candle light.	3·7	8–9	IV	Mulberry red; dull green.
	Jargoon; dirty yellowish	4·5	7–8	II	Dull green; whitish.
BLUE.	Sapphire; pale blue to deep blue	4·0	9	II	Darker blue; paler and greener blue.
	Blue Diamond; pale blue	3·52	10	I	Both alike.
	Blue Topaz; sea blue	3·5	8	IV	Blue; pale yellow green.
	Blue Beryl; greenish blue	2·7	8	III	Blue; green.
	Aquamarine; pale sea blue				Pale blue-green; pale yellowish green.
	Indicolite (Tourmaline); indigo	3·15	7–8	III	Indigo; greyish blue.
VIOLET.	Violet Sapphire (Oriental Amethyst)	4·0	9	II	Violet blue; duller blue.
	Violet Spinel; violet or mauve	3·5	8	I	Both alike.
	Violet Tourmaline; slaty violet	3·15	7–8	III	Violet; grey blue.
	Amethyst; amethyst	2·66	7	III	
	Iolite (Cordierite); dull violet or lavender	2·63	7–7½	IV	Deep blue; buff.
	Kunzite; lilac	3·17	6½–7	V	Lilac; pale lilac.

COLOURS, DENSITY AND HARDNESS OF GEMS—*continued.*

	Name and Tint.	Density.	Hardness.	Crystal System.	Twin Colours seen in Dichroïscope.
BLACK.	Black Diamond	3·52	10	I	Both alike.
	Tourmaline } very dark brown or green . {	3·15	7–8	III	Differently dark.
	Epidote } very dark brown or green . {	3·3	7–6	V	Differently dark.
WHITE STONES.	Diamond	3·52	10	I	Both alike.
	White Sapphire	3·95	9	II	Both alike.
	White Topaz } faint bluish tint . {	3·5	8	IV	Both alike.
	White Beryl } faint bluish tint . {	2·7	7–8	III	Both alike.
	Phenakite	2·97	7–8	III	Both alike.
	Jargoon (White Jacinth); often dirty white.	4·5	7–8	II	Grey and white.
	Andalusite; greyish white or reddish .	3·15	$7\frac{1}{2}$	IV	Blood red; grey white.
	Quartz (Rock Crystal)	2·66	7	III	Both alike.
	White Spinel	3·5	8	I	Both alike.
OPAQUE.	Heliotrope (Bloodstone); dark green with red flecks.	2·6	7		
	Malachite; full green	3·8	$3\frac{1}{2}$		
	Lapis Lazuli; ultramarine	2·4	5		
	Turquoise; sky blue	2·75	6		
	Nephrite (Jade); dull emerald . .	3·33	$6\frac{1}{2}$–7		

Opal has a density of 2·6 and a hardness of 6-7·11.
Noble Opal is transparent, but shot with brilliant streaks of blues, greens and reds. *Fire Opal* is yellowish, shot with red. *Milk Opal* turbid white, shot with red or other tints.
The Chatoyant Gems, such as Cat's Eye, Tiger's Eye (Krokidolite), Moonstone (Felspar) and the like, are readily recognised by their shimmer.

In the above Table the *Density* means the Specific Gravity as compared with water as 1.

The *Hardness* is on the ordinary empirical scale as follows:—10 Diamond; 9 Corundum; 8 Topaz; 7 Quartz; 6 Felspar; 5 Apatite; 4 Fluorspar; 3 Calcspar; 2 Gypsum; 1 Talc.

The *Crystalline Systems* are as follows:—I Cubical; II Tesseral or Square Prismatic; III Hexagonal; IV Right Rhombic or Trimetric; V Monoclinic; VI Triclinic. Of these I has *no* optic axis; II and III have *one* optic axis; while IV, V and VI have *two* optic axes. Those of class I are *monochroic*, all others are *dichroic*. The Dichroïscope affords one of the most useful of tests in distinguishing gems. Thus garnets and spinels are monochroic, and can instantly be distinguished from rubies and rubellites, which are dichroic. *No false gems are dichroic.* Reconstructed rubies are dichroic, being optically identical with the natural gem.

93 TINTS OF NEWTON'S COLOURS OF THIN FILMS (AIR).

Order.	Film Thickness, in millionths of 1 inch.	Tint in Reflected Light.
I.	0	Black.
	3·5	Grey.
	5·5	Whitish.
	8	Straw.
	10	Orange.
	10·5	Brick Red.
	11	Dark Purple.
II.	11·5	Violet.
	13	Blue.
	15	Peacock.
	18	Yellow.
	19·5	Orange.
	21	Red.
	22	Violet.
III.	24	Blue.
	25·5	Peacock.
	27	Green.
	29·5	Yellowish Green.
	31	Rose.
	32·5	Crimson.
	33	Purple.
IV.	34·5	Violet.
	36	Peacock.
	38	Green.
	40	Yellowish Green.
	44	Rose.
V.	48	Pale Green.
	52	Pale Rose.
	55	Rose.
VI.	60	Pale Peacock.
	64	Pale Rose.
	66	Rose
VII.	71	Pale Green.
	74	Pale Rose.

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Liquids

Heptane	1·39.
Carbon tetrachlor	1·46
Nitro benzene	1·55
Bromoform	1·59.

Solids

sodium aluminium alum	1·439
Pot " "	1·456
Pot chloride	1·49
Sod "	1·544
Pot Brom	1·559
Ammon Chlor	1·640
Pot Iodide	1·667
Ammon Iodide	1·700
Cesium Iodide	1·788
Pot Mercuric Iodide	

A B C D E F G H. X

Pot Iodide
Mercury Pot Iod
Zinc Sulph
Ice 1·31
Mica 1'56

Gum Arabic
Sugar 1'56
Sulphur